GLOBAL COLLEGE OF MANAGEMENT

Second Term-2072

Grade: XII **Subject: Basic Mathematics** **F.M.:100**

Time: 3 Hrs. **P.M.: 40**

Set A

Group A **[5×3×2=30]**

Attempt all the questions

1. a. A man has five friends. In how many ways can he invite one or more of them to a dinner.

   b. Prove that:
   $$2\ln x-\ln(x+1)-\ln(x-1)=\frac{1}{x^2}+\frac{1}{2x^4}+\frac{1}{3x^6}+\ldots\ldots\ldots$$

   c. If the binary operation $*$ on Q, the set of rational numbers is defined by $a*b=a+b+ab$ for every $a,b\in Q$ show that $*$ satisfies associative property.

2. a. Find the focus and directrix of the parabola $y^2-4y-8x-20=0$.

   b. Find the ratio in which the line joining the points $(2, 4, 5)$ and $(3, 5, -4)$ is divide by $XY-$ plane.

   c. Find the direction $\cos ine$ of a line which are equally inclined to the axes.

3. a. If $\overrightarrow{3i}+\vec{j}+\vec{k}$ and $\overrightarrow{\lambda i}+\overrightarrow{4j}+\overrightarrow{4k}$ are collinear vectors. Find the value of λ.

   b. Find the area of the parallelogram determined by the vector $\vec{i}+\overrightarrow{2j}+\overrightarrow{3k}$ and $\overrightarrow{-3i}-\overrightarrow{2j}+\vec{k}$.

   c. Evaluate: $\lim_{x\to 0}\frac{\tan x-x}{x-\sin x}$

4. a. Find the slope and inclination with $X-axis$ of the tangent of $x^2+y^2=36$ at $(0, 6)$.

   b. Evaluate: $\int\frac{dx}{\sin x+\cos x}$

   c. Solve: $e^{x-y}\,dx+e^{y-x}\,dy=0$

5. a. The letters are selected at random from the word 'Examination'. Find the probability that both of them are same letters.

   b. Draw the graph of the following inequalities $3x+2y\le 12$, $x,y\ge 0$ shaded the feasible region.

   c. Find the inverse of matrix by using Gauss Jordan method.
   $$\begin{bmatrix}2 & 1\\ 4 & 5\end{bmatrix}$$

Group B **[4×2×5=40]**

6. a. In how many ways can the letters of the words 'COMPUTER' be arranged so that:
   i. all vowels are not always together
   ii. the vowels may occupy only odd position.

   b. If $c_0, c_1, c_2\ldots\ldots\ldots\ldots\ldots c_n$ are the binomial coefficient in the expansion of $(1+x)^n$, prove that
   $$c_0c_1+c_1c_2+c_2c_3+\ldots\ldots\ldots+c_{n-1}c_n=\frac{(2n)!}{(n+1)!(n-1)!}$$

7. a. If $a, b \in (G, \circ)$, then prove that:

i. $(a \circ b)^{-1} \circ = b^{-1} \circ a^{-1}$

ii. $(a^{-1})^{-1} = a$

.

b. What is conic section? A tangent to the parabola $y^2 = 8x$ makes an angle of 45° with the straight line $y = 3x + 5$. Find its equations.

8. a. Find angle between two lines AB and CD whose direction $\cos ines$ are (l_1, m_1, n_1) and (l_2, m_2, n_2) .Also write conditions of perpendicularity and parallelism.

b. Evaluate: $\int \frac{x^2\ dx}{(x^2+4)(x^2+9)}$

9. a. Define order of differential equation.
Solve: $(x^2+1)\frac{dy}{dx} + 2xy = 3x^2$.

b. The mean and variance of binomial distribution are 4 and $4/3$ respectively. Find $p(x \geq 1)$.

10. a. Solve the following system of equations using Gauss- siedel method: $2x - y = 1,\ 3y - x - z = 8,\ y - 2z = 5$.

b. Solve the following system of equations using Guassion elimination method. (if possible)

$$x_1 - x_2 + x_3 = 1$$
$$3x_1 + x_2 + 5x_3 = 11$$
$$4x_1 + 2x_2 + 7x_3 = 16$$

Group C [6×5=30]

11. Prove that the tangents and normal at the ends of the latus rectum of a parabola form the sides of a square.

12. Define plane. Show that $ax + by + d = 0$ represents a plane parallel to $z - axis$. Find the equations of the plane through the points $(1, 2, 3)$ and $(7, 5, 6)$ parallel to $x - axis$.

13. Define scalar product of two vectors. Find the geometrical interpretation of scalar product of two vectors. Prove vectorically that $\cos(A+B) = \cos A.\cos B - \sin A.\sin B$.

14. State Rolle's Theorem of differential calculus and interpret it geometrically. Verify that Rolle's theorem for the function $f(x) = x^2 - 5x + 4$ in $[1, 4]$

15. Minimize $w = 18x + 12y$ subject to the constraints

$2x + y \geq 8$

$6x + 6y \geq 36$

where $x, y \geq 0$

(By using simplex method)

Best of Luck

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Set B

Group A **[5×2×3=30]**

Attempt all the questions

1. a. In an examination, a student has to pass in each of the 4 subjects. In how many ways can he fail.

   b. If $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4}$............. show that $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + ...$

   c. Let $G = \{1, w, w^2\}$ $'\times'$ denote the usual operation of multiplication and w represent cube roots of unity. Show that an operation $'\times'$ is binary operation.

2. a. Find the vertex and focus of the parabola $y^2 = 4x + 4y$.

   b. Find the unit vector perpendicular to each of the given pair of vectors $(4, -2, 3)$ and $(5, 1, -4)$

   c. If $\propto, \beta$ and γ are the angles which a line makes with the co-ordinate axes. Prove that $\sin^2 \propto + \sin^2 \beta + \sin^2 \gamma = 2$.

3. a. If $\vec{a} + \vec{b} + \vec{c} = 0$. $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 7$ find the angle between $\vec{a}$ and $\vec{b}$.

   b. Find the ratio in which the $YZ -$ plane divides the line joining $(4, 6, 7)$ and $(-1, 2, 5)$. Also find the coordinate of the point on the $YZ -$ plane .

   c. Evaluate: $\lim_{x \to \pi/2} \left(\frac{\sec 3x}{\sec x} \right)$.

4. a. .Find the angle of intersection between the curve $y = x^2$ and $6y = 7 - x^2$ at $(1, 1)$.

   b. Evaluate: $\int \text{cosec}x \; dx$

   c. Solve: $\sqrt{1-x^2} \; dy + \sqrt{1-y^2} \; dx = 0$

5. a. Given $P(A) = 0.4$, $P(A \cup B) = 0.56$ and $P(B) = 0.3$. Are A and B independent.

   b. Shade the feasible region determined by the inequalities $x + 2y \le 10$, $x + y \le 6$, $x, y \ge 0$

   c. Find the inverse of a matrix by using Gauss Jordan method $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$.

Group B **[4×2×5=40]**

6. a. Prove that the total number of permutation of a set of n object taker r at a time is given by $p(n, r) = \frac{n!}{(n-r)!}$.

   b. If $(1+x)^n = c_0 + c_1 x + c_2 x^2 + ... + c_n x^n$ prove that $c_0^{\,2} + c_1^{\,2} + c_2^{\,2} + ... + c_n^{\,2} = \frac{(2n)!}{(n!)^2}$.

7. a. Let $G = Q - \{1\}$, the set of all rational numbers without the unit number. Suppose an operation $*$ defined on G is given by $(a * b) = a + b - ab$. Show that the system is a group.

b. Find the equation of the tangents from the point $(-6, 9)$ to the parabola $y^2 = 24x$.

8. a. Prove that the two lines whose direction cosines are given by $l + m + n = 0$ and $2mn + 3nl - 5lm = 0$ are perpendicular to each other.

b. Evaluate: $\int \frac{dx}{1 - 2\cos x}$

9. a. Define degree of differential equations.

Solve: $\tan x.\frac{dy}{dx} + y = \sec x$.

b. State and prove theorem of total probability.

10. a. Solve the following system of equations using Gauss – siedal method:

$3x + 12y - z = 28, \quad x + 4y + 7z = 2, \quad 10x + 4y - 2z = 20$

b. Using Gauss elimination method. Solve the following system of equations: $x - 2y + 3z = 2, \quad 2x - 3y + z = 1, \quad 3x - y + 2z = 9$

Group C **[6×5=30]**

11. What is conic section? Find the condition under which the line $y = mx + c$ tangent to the parabola $y^2 = 4ax$. Find the equation of tangent in slope form. Also, find the point of contact.

12. Find the equation of the plane through the points $(2, 2, 1)$ and $(9, 3, 6)$ and normal to the plane $2x + 6y + 6z = 9$. Also, find the equation of the plane through the point $(1, 1, 0)$, $(-2, 2, -1)$ and $(1, 2, 1)$.

13. Define cross product of two vector. Also, find the geometrical interpretation of cross product of two vectors. Prove vecterically that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$.

14. State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x) = x(x-1)^2$ in $[0, 2]$.

15. Minimize $w = 3x + 3y$ subject to the constraints

$2x + y \geq 4$

$x + 2y \geq 4$

where $x \geq 0, \quad y \geq 0$

(By using simplex method)

Best of Luck