Pre-Board Exam – 2072

Grade: XII | Subject: Basic Mathematics | F.M.:100

Time: 3 Hrs. | P.M.: 35

Set 'A'

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

1. a) It is required to seat 5 boys and 4 girls in a row so that the girls occupy the even places. How many such arrangements are possible? [2]

   b) Prove that $\log_e 2 = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \ldots$ [2]

   c) Let $a * b = 3a + 2b$ for $a, b \in Z$. Is $*$ commutative binary operation on Z? [2]

2. a) Find the eccentricity and the foci of the hyperbola $3x^2 - 4y^2 = 36$. [2]

   b) Find the ratio in which the line joining the points $(-2, 4, 7)$ and $(3, -5, -8)$ is divided xy-plane. [2]

   c) Find the area of the triangle determined by the vectors $3\vec{i} + 4\vec{j}$ and $-5\vec{i} + 7\vec{j}$. [2]

3. a) Find the derivative of $(\ln x)^{\sinh x}$ [2]

   b) Find the integral: $\int \frac{dx}{\sqrt{2ax - x^2}}$ [2]

   c) Find the integral: $\int \frac{dx}{1 + \sin x}$ [2]

4. a) Solve: $\frac{dy}{dx} = \frac{x - y + 1}{x + y + 1}$ [2]

   b) The covariance between the variable x and y is 18 and the variance of x and y are 16 and 81 respectively. Find the coefficient of correlation between them. [2]

   c) Given $P(A) = 0.4$, $P(A \cup B) = 0.56$, $P(B) = 0.3$. Are A and B independent? [2]

5. a) In how many ways can the letters of the word SUNDAY be arranged. How many of these arrangements do not begin with S? How many begin with S and do not end with Y? [4]

   OR

   From 6 gentlemen and 4 ladies a committee of 5 is to be formed. In how many ways can this be done so as to include at least one lady? [4]

   b) What is group? Prove that if every element of a group G is its own inverse, then G is an abelian. [4]

6. a) Find the integral: $\int \frac{x}{x^3 + 1}\, dx$ [4]

   b) Solve: $\tan x \frac{dy}{dx} + y = \sec x$. [4]

7. a) State mean value theorem. Verify mean value theorem for the function $f(x) = x^3 + x^2 - 6x$ in $[-1, 4]$. [4]

   OR

   Find from the first principle the derivatives of: $\ln(\tan^{-1} x)$[4]

   b) Five men in a group of 20 are graduates. If three men are chosen out of 20 at random what the probability is of at least one being graduates. [4]

8. a) Prove that the line $lx + my + n = 0$ will be a normal to the parabola $y^2 = 4ax$ if $al(2m^2 + l^2) + m^2 n = 0$. [4]

OR

What is a conic section? Find the eccentricity, the coordinates of the vertices and the foci of ellipse $9x^2 + 5y^2 - 30y = 0$. [4]

b) Prove by vector method: In any triangle ABC $\cos(A + B) = \cos A \cos B - \sin A . \sin B$ [4]

9. If $(1 + x)^n = C_0 + C_1 x + C_2 x^2 + \ldots + C_n x^n$, prove that $C_0 C_2 + C_1 C_3 + C_2 C_4 + \ldots + C_{n-2} C_n = \dfrac{2n!}{(n-2)!(n+2)!}$. Also find the 7th term in the expansion of $\left(2x + \dfrac{1}{y}\right)^{10}$. [6]

10. Derive the angle between two lines whose dc's are (l_1, m_1, n_1) and (l_2, m_2, n_2). Also derive the condition of perpendicularity and parallism. [6]

OR

Define plane. A variable plane is at a constant distance 3p from the origin and meets the axes in the points A, B, C. Prove that the locus of the centroid of the triangle ABC is $\dfrac{1}{x^2} + \dfrac{1}{y^2} + \dfrac{1}{z^2} = \dfrac{1}{p^2}$. [6]

11. From the following data between the ages of husbands and wife's. Calculate the two regression equations and find the husband's age when wife's age is 20 and wife's age when husband's age is 30. [6]

Wife's age (X)	18	20	22	23	27	28	30
Husband's age (Y)	23	25	27	30	32	31	35

Group C

12. a) Find all basic feasible solutions of the given system of equations $2x + y = 3$; $x - y + z = 2$. [2]

b) Convert the decimal numerals 86.25 into octal form. [2]

c) Solve the given system of equation by Gauss-Seidel method: $5x - 2y = 7;\ 2x + 3y = 18$ [2]

13. a) Rank the given matrix according to their conditioned number

$$\begin{bmatrix} 1 & 8 & -1 \\ -9 & -71 & 11 \\ 1 & 17 & 18 \end{bmatrix}$$ [4]

OR

Solve the given system of equation by using Gauss elimination method.
$x + 2y + 3z = 2;\ x + y - z = 1;\ 2x + 3y + 2z = 3$ [4]

b) Minimize $W = 2x + y$ Subject to
$5x + y \geq 9;\ 2x + 2y \geq 10;\ x \geq 0;\ y \geq 0$
by using simplex method [4]

14. Apply the method of successive bisection to find the root of the equation $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to two places of decimal by successive bisection method. [6]

OR

Find Newton's method to find the positive root of $Sinx + x - 1 = 0$ in (0, 1). [6]

15. Determine using a) Trapezoidal rule b) Simpson's rule, the following integrals. Estimate the error in each case $\int_1^3 \dfrac{dx}{x^2}$, $n = 4$.
Also estimate the error in using the approximation $n = 4$ from its actual value. [6]

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GLOBAL COLLEGE OF MANAGEMENT

Pre-Board Exam – 2072

Grade: XII — Subject: Basic Mathematics — F.M.:100

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Set 'B'

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Group A

1. a) How many even numbers of 3 digits can be formed if the repetition of digits is allowed? [2]

   b) Prove that $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\ldots=1-\ln 2$ [2]

   c) Let G be the set of subsets of the set $\{0, 1\}$. Show that the set G is closed under the operation of union. [2]

2. a) Determine the equation of the hyperbola with a focus at $(6, 0)$ and vertex at $(4, 0)$. [2]

   b) If O is the origin and $P(2, 3, 4)$ and $Q(1, -2, 1)$ be any two points, show that OP is perpendicular to OQ. [2]

   c) Prove vectorically that $a^2 = b^2 + c^2 - 2bc \cos A$ [2]

3. a) Find the derivative of $x^y = y^x$ [2]

   b) Find the integral: $\int \frac{dx}{(11+x)\sqrt{2+x}}$ [2]

   c) Find the integral: $\int \frac{dx}{x+\sqrt{x^2-1}}$ [2]

4. a) Solve: $\frac{dy}{dx}=\frac{xy+y}{xy+x}$ [2]

   b) In a certain distribution the following results were obtained: Mean = 45, median = 48, coefficient of skewness = – 0.4. Find the standard deviation and the coefficient of variation. [2]

   c) Find the probability of getting two heads twice in 5 tosses of two coins. [2]

5. a) How many different permutations can be made with letters of the word RANDOM under the following conditions

   i) if permutations begin with D

   ii) if permutations end with N

   iii) if permutations begin with A and end with O

   iv) if vowels are never separated [4]

   OR

   A committee of 5 is to be formed from 6 gentlemen and 4 ladies. In how many ways can it be done when

   i) At least two ladies are included?

   ii) At most two ladies are included? [4]

   b) If a and b are the elements of group $(G, 0)$ then $a\,0\,x = b$ have unique solution in $(G, 0)$. [4]

6. a) Find the integral: $\int \frac{1-x}{x^2+x^3}\,dx$ [4]

   b) Solve: $(1+x^2)\frac{dy}{dx}+y=e^{\tan^{-1}x}$ [4]

7. a) State Rolle's theorem. Verify Rolle's theorem for the function $f(x)=\sin 2x$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. [4]

   OR

   Find from the first principle the derivatives of: $\cos(\ln x)$ [4]

   b) In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the probability p of a success in a single trial. [4]

8. a) Find the area of the triangle formed by the lines joining the vertex of the parabola $y^2 = 12x$ to the ends of its latus rectum. [4]

OR

Find the vertices, eccentricities, foci and length of major axis and minor axis of the ellipses

$9x^2 + 4y^2 + 40y + 18x + 73 = 0$. [4]

b) Prove by vector method: In any triangle ABC

$Sin(A - B) = SinA.CosB - CosA.SinB$ [4]

9. If $(1+x)^n = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n$, prove that $C_0 + 3C_1 + 5C_2 + \ldots + (2n+1)C_n = (n+1)2^n$. Also find the middle term in the expansion of $\left(ax - \frac{1}{ax}\right)^{2n}$. [6]

10. Define direction cosines and direction ratios of a line. Find the direction cosines of two lines which satisfy the relations $2l + 2m - n = 0$ and $lm + mn + nl = 0$. Also find the angle between the two lines. [6]

OR

Find the length of perpendicular from a given point on a given plane $lx + my + nz = p$. Also find the length of the perpendicular from the point $(2, 3, 4)$ on the plane $3x - 2y + 6z + 4 = 0$. [6]

11. Marks of two students in six examinations out of total score 100 were as follows.

Student A	40	60	70	80	50
Student B	60	30	80	70	60

Find which student may be considered to be more consistent. [6]

Group C

12. a) Test the consistency of the given system of equations:

$-2x + y + 3z = 12;\ x + 2y + 5z = 4;\ 6x - 3y - 9z = 24$ [2]

b) Write down the initial tableau and find the initial solution of the given L.P problem Max. $Z = 7x + 5y$ Subject to

$x + 2y \le 6;\ 4x + 3y \le 6;\ x \ge 0;\ y \ge 0$ [2]

c) Convert the hexadecimal numeral AFB2 to binary form. [2]

13. a) Solve the given system of equations by Gauss-Seidel method.

$5x + 3y - z = 15;\ 2x - 4y + z = -4;\ x - y - 3z = -13$ [4]

OR

Solve the system of equations by using Gauss Jordan method

$2x - y + z = -2;\ x + y - 2z = -9;\ x + 2y + z = 9$ [4]

b) Using simplex method, minimize $W = 2x + 2y$

Subject to

$x + 2y \ge 3$

$3x + 2y \ge 5$ [4]

$x \ge 0,\ y \ge 0$

14. Use Newton-Raphson's method to approximate $\sqrt[3]{2}$ with an error less than 0.00001. [6]

OR

Determine the number of iterations required by bisection method necessary to solve $f(x) = x^3 + x - 4 = 0$ in the interval [1, 4] within an accuracy 10^{-2}. Find a root of the equation within an accuracy of 10^{-1}. [6]

15. Estimate the integrals $\int_0^3 x\,dx$ taking n = 6 sub intervals by Trapezoidal and Simpson's rule. Estimate the error in each case. Also estimate the error in using the approximation n = 6, from its actual value. [6]

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