



FIRST TERM EXAM-2072

Grade: XII
Time: 3:00 hrs.

Subject: Basic Mathematics

F.M.:100
P.M.:40

Set A

Attempt all the questions:

Group-A
[5×3×2 = 30]

1. a. In how many ways can 5 boys and 5 girls be seated in a row so that
 - (i) all girls sit together.
 - (ii) No two girls are sit together.
 - b. If the coefficient of x in the expansion of $\left(x^2 + \frac{a}{x}\right)^5$ is 270, find a .
 - c. Prove that $\frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots = \ln 2$.
2. a. Find the equation of parabola in which the ends of the latus rectum $(-1, 5)$ and $(-1, 11)$ and the vertex is $(-5, -3)$.
 - b. Find the unit vector perpendicular to each of the vector $3\vec{i} + \vec{j} + 2\vec{k}$ and $2\vec{i} - 2\vec{j} + 4\vec{k}$
 - c. At what angle does the curve $y(1+x)=x$ cut the x -axis.
3. a. Find the derivative of $x^{\cosh^2 \frac{x}{a}}$.
 - b. Using L- Hospital' rule evaluate: $\lim_{x \rightarrow 0} \frac{e^x + e^{-x} - 2 \cos x}{\sin^2 x}$
 - c. Find the ratio in which the line joining the points $(2, 4, 5)$ and $(3, 5, -4)$ is divide by XY - plan.

4. a. For what value of m are the vectors $\vec{i} + \vec{j} - 4\vec{k}$ and $2\vec{i} - 7\vec{j} + m\vec{k}$ perpendicular?
 - b. Evaluate: $\int \frac{dx}{\sqrt{2ax + x^2}}$
 - c. Evaluate: $\int \frac{1}{x(x^n + 1)} dx$.
5. a. Solve: $\sqrt{1-x^2} dy + \sqrt{1-y^2} dx = 0$
 - b. Solve: $\frac{dy}{dx} + 4x = 2e^{2x}$
 - c. Two dice are thrown together. Find the probability of a sum ≤ 5 .

Group-B
[5×2×4=40]

6. a. In how many ways can the letters of the word "TUESDAY" be arranged? How many of these arrangements do not begin with T. How many begin with T and do not end with Y.
 - b. Prove that : $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \dots = \frac{3e}{2}$
7. a. Find the vertices, eccentricities, foci and latus rectum of the hyperbola. $16x^2 - 9y^2 + 96x - 72y + 144 = 0$
 - b. Prove that the line $3x+4y+6=0$ is tangent to the parabola $2y^2=9x$. find the point of contact.

- 8 a. Find, from first principle the derivative of $\log\left(\cos\frac{x}{a}\right)$.
- b. If \vec{a} and \vec{b} are two vectors of unit length and θ be the angle between them, show that $\frac{1}{2}|\vec{a} - \vec{b}| = \sin\frac{\theta}{2}$. Also prove by vector method : $b=c \cos A + a \cos C$.
- 9 a. Evaluate: $\int \frac{dx}{1-3\sin x}$.
- b. Solve: $\sin x \frac{dy}{dx} + (\cos x).y = \sin x . \cos x$.
- 10 a. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the probability p of a success in a single trial.
- b. A bag contain 5 red and 6 white balls. Two balls are drawn at random. Find the probability that
- both are of the same colour.
 - one is red.

Group-C
[5×6 = 30]

- 11 Prove that the sum of the coefficients of the odd terms in the expansion of $(1+x)^n$ is equal to the sum of the coefficient of the even terms and each is equal to 2^{n-1} . If $(1+x)^n = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$; prove that:

$$c_0c_2 + c_1c_3 + c_2c_4 + \dots + c_{n-2}c_n = \frac{(2n)!}{(n-2)!(n+2)!}$$
- 12 What are the direction cosines of a line? Prove that $\sin^2 \alpha + \sin^2 \beta + \sin^2 \gamma = 2$. Also find the angle between the lines whose direction cosines l, m, n satisfies the equation: $l+m+n=0$ and $2lm+2ln-mn=0$
- 13 State Rolle's Theorem. Interpret it geometrically. Verify Rolle's Theorem for the function $f(x)=(x-3)^2$ for $x \in [0,3]$
- 14 A committee of 5 is to be selected from 6 boys and 5 girls. Determine the number of ways of selection the committee. If the committees consist of at least 1 boy and 1 girl. Also prove that: $c(n, r)+c(n, r-1) = c(n+1, r)$.
- 15 Define dot product of two vectors and interpret it geometrically. Prove that: $\sin(A-B) = \sin A . \cos B - \cos A . \sin B$



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Set B

Group-A

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Attempt all the questions:

- 1 a. There are 5 boys and 3 girls. In how many way can they stand in a row so that : (i) they may stand anywhere
(ii) no two girls are together
- b. If the term independent of x in the expansion of $\left(\sqrt{x} - \frac{a}{x^2}\right)^{10}$ is 405, find a .
- c. Prove that : $\frac{2}{3!} + \frac{4}{5!} + \frac{6}{7!} + \dots = \frac{1}{e}$
- 2 a. Find the coordinate of the vertex and focus of the parabola whose equation is $y^2=6y - 12x + 45$.
- b. If $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, prove that \vec{a} is perpendicular to \vec{b} .
- c. find the slope and inclination with x -axis of the tangent of $x^2+y^2=36$ at (0, 6)
- 3 a. Find the derivative of $\left(\cosh \frac{x}{a}\right)^{\ln x}$.
- b. Using L- Hospital rule evaluate: $\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$.

- c. Find the ratio in which the YZ- plane divides the join of the points (-2, 4, 7) and (3, -5, 8).

- 4 a. Find the area of parallelogram whose diagonals are represented by vectors $3\vec{i} - \vec{j} - 2\vec{k}$ and $\vec{i} - 3\vec{j} + 4\vec{k}$

- b. Integrate: $\int \frac{dx}{\sqrt{2ax - x^2}}$

- c. Evaluate : $\int \frac{1}{x(x^n + 1)} dx$

- 5 a. Solve: $x^2 dy - y^2 dx = 0$

- b. Solve: $\frac{dy}{dx} = \cos(x + y)$

- c. Two dice are thrown once. Determine the probability of getting a sum ≤ 3 .

Group-B

[5×2×4=40]

- 6 a. In how many ways can the letters of the word GLOBAL be arranged. How many of these arrangements do not begin with G. how many begin with G and end with L.
- b. Prove that : $\frac{1}{2!} + \frac{1+2}{3!} + \frac{1+2+3}{4!} + \dots = \frac{e}{2}$
- 7 a. Deduce the equation of the tangent to the parabola $y^2=4ax$ at point (x_1, y_1)

b. Find the vertices, eccentricity, length of latus rectum and foci of the ellipse $\frac{x^2}{8} + \frac{(y-2)^2}{12} = 1$

Group-C
[5×6 = 30]

- 8 a. Find from first principle the derivative of: $\log\left(\sin\frac{x}{a}\right)$.
- b. In any triangle: $\vec{a} + \vec{b} + \vec{c} = 0$ $|\vec{a}| = 3, |\vec{b}| = 5, |\vec{c}| = 7$, find the angle between \vec{a} and \vec{b} also prove by vector method $a = b \cos C + c \cos B$

- 9 a. Evaluate: $\int \frac{dx}{1-2\cos x}$
- b. Solve: $(1-x^2)\frac{dy}{dx} = 1+xy$.

- 10 a. In a binomial distribution consisting of 5 independent trials, the probability of 1 and 2 successes are 0.4096 and 0.2048 respectively. Find the probability p of a success in a single trial.
- b. A bag contains 6 white balls and 8 blue balls. Two balls are drawn from the bag at random one after another without replacement. Find the probability:
- the first is white and second is blue.
 - both are white.

- 11 Prove that the sum of the coefficients of the odd terms in the expansion of $(1+x)^n$ is equal to the sum of the coefficient of the even terms and each is equal to 2^{n-1} .
If $(1+x)^n = c_0 + c_1x + c_2x^2 + \dots + c_nx^n$;
prove that: $c_0^2 + c_1^2 + c_2^2 + c_3^2 + \dots + c_n^2 = \frac{(2n)!}{(n!)^2}$.

- 12 Define direction cosines of a line? Prove that $l^2 + m^2 + n^2 = 1$.
Where l, m, and n are direction cosines. Also find the direction cosines l, m, n of two lines which are connected by the relations:
 $4l+3m-2n = 0, lm-mn+nl=0$

- 13 State Mean value theorem, Interpret it geometrically. Verify mean value theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[1, 4]$

- 14 Define cross product of two vectors and interpret it geometrically also prove by vector method
 $\cos(A-B) = \cos A \cos B + \sin A \sin B$.

- 15 A candidate is required to answer 6 out of 10 questions which are divided into two groups each contain 5 questions and he is not permitted to attempt more than 4 from any group. In how many ways can he make up his choice.
Also find the number of ways in which 5 courses out of 8 courses can be selected when 3 courses compulsory.
