**Second Term Exam - 2071**

Grade: XII | Subject: Basic Mathematics | F.M.: 100
Time: 3:00 Hrs. | | P.M.: 40

**Set A**

**Group A** **[5×3×2=30]**

1. a) In how many ways can the letters of the word "CALCULUS" be arranged so that the two C's do not come together.

   b) Find the constant term in the expansion of $\left(\frac{4x^2}{3}-\frac{3}{2x}\right)^9$.

   c) Given an algebraic structure $(Z, \bullet)$, with the binary operation $\bullet$ define by $m \bullet n = m + n - 1$ for all $m, n \in Z$. Determine the identity element and inverse element of $-2$.

2. a) Find the equation of the parabola with vertex at (–1, 1) and directrix $y = 3$.

   b) Find the ratio in which the line joining the points (–2, 4, 7) and (3, –5, –8) is divided by the $xy$ – plane.

   c) Find the area of the parallelogram whose three of four vertices are (1, 1, 2), (2, –1, 1) and (3, 2, –1).

3. a) If $\vec{a} = (1, 2, 3)$ and $\vec{b} = (2, 3, 4)$ find the projection of $\vec{a}$ on $\vec{b}$.

   b) Use differentials to approximate the change in $x^3$ as $x$ change from 5 to 5.01.

   c) Find the equation of the normal to the curve: $y = x^3 - 2x^2 + 4$ at (2, 4).

4. a) Evaluate: $\int \frac{x}{\sqrt{a^4+x^4}}\, dx$

   b) Solve: $\sec^2 x \tan y\, dx + \sec^2 y \tan x\, dy = 0$

   c) A coin is tossed successively three times. Find the sample space and determine the probability of getting at least 2 heads.

5. a) Graph the given system of inequalities and find the vertices, if they exist. $3x + y \le 3$; $4 - y \le 2x$

   b) Using Gauss-Jordan method, find the inverse of the given matrix $\begin{bmatrix} 5 & 4 \\ 3 & 2 \end{bmatrix}$.

   c) Evaluate, using simpson's rule the integral $\int_0^1 \frac{dx}{1+x}$, for $n = 4$.

**Group B** **[5×2×4=40]**

6. a) In how many ways can the letters of the word "COMBINE" be arranged so that

   i) the vowels are never separated

   ii) all the vowels are never come together

   iii) vowels occupy only the odd places

   b) Show that the middle term of the expansion of $(1+x)^{2n}$ is $\frac{1.3.5...(2n-1)}{n!}2^n x^n$

7. a) Define group. If $a$ and $b$ are the elements of a group $(G, 0)$, then $a\ 0\ x = b$ have unique solutions in $(G, 0)$.

   b) Find the direction cosine of the line which is perpendicular to the lines with direction cosines proportional to 3, –1, 1 and –3, 2, 4.

8. a) Integrate: $\int \frac{x^3}{2x^4 - 3x^2 - 5}\, dx$

b) Solve: $(x+1)\frac{dy}{dx} + 2y = \frac{e^x}{(x+1)}$

9. a) Suppose that in a certain city 60% of all recorded births are males. Suppose we select 5 birth records from population. What is the probability that

i) less or equal to 2 are males

ii) more than 4 are males

b) Solve the given systems of equations using Gauss elimination method.

$x - y + z = 1;\ 3x + y + 5z = 11;\ 4x + 2y + 7z = 16$

10. a) Use the Gauss-Seidal method to solve the system $2x - y = 1;\ 3y - x - z = 8;\ y - 2z = 5$

b) Evaluate using trapezoidal rule to integral $\int_0^\pi \sin x\, dx$.

Estimate the trapezoidal error if $n = 4$.

**Group C** **[6×5=30]**

11. Define conic section. Find the vertices, eccentricity, foci and latus rectum of the hyperbola, $16x^2 - 9y^2 + 96x - 72y + 144 = 0$.

12. Define plane. Prove that the general equation of the first degree in *x*, *y* and *z* represent a plane.

13. Define linearly dependent and linearly independent vector. Examine whether the given vectors are linearly dependent or independent. $\vec{a} = (1, -2, 1)$, $\vec{b} = (2, 1, -1)$ and $\vec{c} = (7, -4, 1)$.

14. Find, from first principle, the derivative of $\log(\sec x^2)$.

15. Use the simplex method to find the optimal solution of the given L.P problem: Max. $Z = 5x + 3y$

Subject to $2x + y \le 40$; $x + 2y \le 50$; $x, y \ge 0$.

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**Set B**

**Group A** **[5×3×2=30]**

1. a) In how many ways can the letters of the word "ARRANGE' be arranged so that the two R's do not come together.
   b) In the expansion of $(1+x)^{43}$ coefficients of $(2r+1)^{th}$ term and $(r+2)^{th}$ term are equal. Find $r$.
   c) Given an algebraic structure $(Z, \bullet)$, with the binary operation $\bullet$ define by $m \bullet n = m + n - 1$ for all $m, n \in Z$. Determine the identity element and inverse element of 3.

2. a) Find the equation of the parabola with vertex at (–1, 2) and directrix $x = 4$.
   b) Find the ratio in which the *yz*-plane divides the line joining the point (4, 6, 7) and (-1, 2, 5).
   c) If $\vec{a} = (1, 2, 3)$ and $\vec{b} = (2, 3, 4)$ find the projection of $\vec{b}$ on $\vec{a}$.

3. a) Find the unit vector perpendicular to each of the vector (2, –1, 1) and (3, 4, –1).
   b) Evaluate: $\lim_{x\to 0} \frac{xe^x - \log(x+1)}{x^2}$
   c) Find the points on the curve $y = x^3 - 3x^2 + 1$ where the tangents are parallel the *x*-axis.

4. a) Prove that: $\int \operatorname{cosec} x\, dx = \log\left(\tan\frac{x}{2}\right) + C$
   b) Solve: $\tan x\, dy + \tan y\, dx = 0$
   c) What is the probability of drawing a heart or an ace from a deck of 52 cards.

5. a) Graph the given system of inequalities and find the vertices if they exist. $x + 2y \ge 0;\ \ 2x + y \le 4$
   b) Using Gauss-Jordan method, find the inverse of the given matrix $\begin{bmatrix} 2 & -1 \\ -3 & 3 \end{bmatrix}$.
   c) Evaluate, using simpson's rule the integral $\int_1^3 \frac{dx}{x^2}$ ,for $n = 4$.

**Group B** **[5×2×4=40]**

6. a) In how many ways can the letters of the word "PETROL" be arranged. How many of these do not begin with P? Also find the number of words which can be formed if O and L are never together.
   b) Find sum to infinity the given series $1 + \frac{3}{1!} + \frac{5}{2!} + \frac{7}{3!} + \ldots$

7. a) Define group. If $a$ and $b \in (G, 0)$, then prove that
      i) $(a\ 0\ b)^{-1} = b^{-1}\ o\ a^{-1}$
      ii) $(a^{-1})^{-1} = a$
   b) The projection of a line on the axes are 6, 2, 3. Find the length of the line and its direction cosines.

8. a) Integrate: $\int \frac{2x^3 + 8x}{2x^4 - 5x^2 + 3}\, dx$
   b) Solve: $(1 + x^2)\frac{dy}{dx} + 2xy = 4x^2$

9. a) Suppose 3 cards are drawn from a well shuffled deck of 52 cards. What is the probability of getting (i) all three diamond (ii) two kings.

b) Solve the given systems of equations using Gauss elimination method.

$x+3y+4z=8;\ 2x+y+2z=5;\ 5x+2z=7$

10. a) Use the Gauss-Seidal method to solve the system $2x+3y-z=9;\ x-y+z=9;\ 3x-y-z=-1$

b) Evaluate using trapezoidal rule to integral $\int_0^{\pi}\cos x\ dx,\ n=6$.

Estimate the trapezoidal error if $n = 6$.

## Group C [6×5=30]

11. Define conic section. Find the vertices, eccentricity, foci and latus rectum of the ellipse, $16x^2+25y^2+64x+50y-311=0$.

12. Define direction cosines of a line. Find the angle between two lines whose direction cosines are $(l_1, m_1, n_1)$ and $(l_2, m_2, n_2)$. Also deduce the condition of parallelism.

13. Define cross product of two vector. Interpret it geometrically. Also prove vectorically that . $\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$

14. Find, from first principle, the derivative of $\log\left(\operatorname{cosec} x^2\right)$.

15. Use the simplex method to find the optimal solution of the given L.P problem: Max. $Z=8x+36y$

Subject to $2x+6y\le 18$; $x+6y\le 12$; $x,\ y\ge 0$.

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