GLOBAL COLLEGE OF MANAGEMENT

Pre-Board Exam – 2071

Grade: XII Subject: Basic Mathematics F.M.: 100

Time: 3 hrs. P.M.: 40

Set A

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group A

1. a) In how many ways can eight people be seated in a row of eight seats so that two particular persons are always together? [2]

   b) Prove that: $\frac{1}{1.2}+\frac{1}{3.4}+\frac{1}{5.6}+.............=\ln 2$ [2]

   c) If $a*b=a$, then prove that $b=e$ [2]

2. a) Prove that line $lx+my+n=0$ touches the parabola $y^2=4ax$ if $l\,n=am^2$ [2]

   b) Find the ratio in which the yz plane divides the line joining the point $(-2,4,7)$ and $(3,-5,8)$ [2]

   c) If $\overrightarrow{3i}+\vec{j}-\vec{k}$ and $\overrightarrow{\lambda i}-\overrightarrow{4j}+\overrightarrow{4k}$ are collinear vectors. Find the λ. [2]

3. a) Find the derivative of : $x^{\cosh x/4}$. [2]

   b) Integrate: $\int\frac{dx}{\sqrt{2ax+x^2}}$ [2]

   c) Solve: $x^2dy-y^2dx=0$ [2]

4. a) From a group of 10 times, $\sum x=452$, $\sum x^2=24270$ and mode= 43.7 find the pearsons coefficients of skewness. [2]

   b) Find the probability of drawing a heart or a king from a deck of 52 cards. [2]

   c) Find the binomial distribution having mean = 12 and variance = 8. [2]

5. a) From 10 football players in how many ways can a selection of 4 be made:

   i. When one particular player is always included.

   ii. When two particular players are always excluded. [2]

   b) If $(1+x)^n=c_0+c_1x+c_2x^2..........+c_nx^n$, prove that $c_0c_n+c_1c_{n-1}........+c_nc_0=c(2n,n)=\frac{(2n)!}{(n!)^2}$ [4]

6. a) Let $*$ be defined on Q^+ by $a*b=ab/2$. Show that $(Q^+,*)$ is an abelian group. [4]

   b) Deduce the equation of the tangent to the parabola $y^2=4ax$ at the point (x_1,y_1). [4]

7 a) Find the derivative of $\sin(\ln x)$ from first principle. [4]

   OR

   State mean value theorem. Interpret it geometrically. Verify mean value theorem for the function $f(x)=(x-1)(x-2)(x-3)$ in $[1,4]$.

   b) Integrate: $\int\frac{dx}{1+2\sin x}$. [4]

   OR

   Integrate: $\int\frac{x^2}{(x+2)(x+3)^2}\,dx$

8. a) Solve: $(x^2+y^2)\,dy=xy.dx$. [4]

   OR

Solve: $Sinx\frac{dy}{dx}+(\cos x)y=\sin x.\cos x.$

b) The probability of maris hitting a target is $\frac{1}{4}$. If he tires 5 times, what is the probability of his hitting the target:

i. exactly thrice

ii. at least thrice [4]

9. Find the angle between the lines whose direction cosine $l+m+n=0$ and $2lm+2l\,n-mn=0$. [6]

OR

Find the equation of the plan through the points $(2,2,1)$ and $(9,3,6)$ and normal to the plane $2x+6y+6z=9$, also find the equation of the plan through $(1,2,3)$ and parallel to the plan $3x-4y+5z=0$.

10. Define vector product of two vectors. Interpret it geometrically. Prove by vector method $\frac{SinA}{a}=\frac{SinB}{b}=\frac{SinC}{c}$. [6]

11. The equation of two regression lines are $4x-5y+33=0$ and $20x-9y=107$, find

i. The mean of x and mean of y.

ii. The regression coefficients

iii. The correlation coefficient between x and y.

iv. The ratio of standard deviation of x and y. [6]

Group C

16. a) Use the Trapezoidal Rule to approximate the integral $\int_1^2\frac{1}{x}dx$. Find the error for the approximation. [2]

b) Find the inverse of the given matrix by Gauss Jordan method. $A=\begin{bmatrix}2 & -1\\ -3 & 3\end{bmatrix}$ [2]

c) Convert the given binary numerals into hexadecimal form. 1010111010. [2]

17. a) Using Newton - Raphon's method to find the positive root of $x^3-18=0$ in (2, 3). [4]

b) Solve the given equations by Gauss elimination method: [4]

$3x-y+z=-2$; $x+5y+2z=6$; $2x+3y+z=0$

OR

Use the Gauss-Siedel method to solve the system:

$3x+y=5$

$x-3y=5$

18. A carpenter has 60, 40 and 25 metres of teak, plywood and rosewood respectively. The product A requires 2, 1, 2 metres and the product B requires 3, 2, 1 metres of teak, plywood and rosewood respectively. If A would sell for Rs. 2560 and B would sell for Rs. 1840 per unit. Give a mathematical formulation of the above LP problem for the maximum income. And find the maximum income of the objective function for a given feasible region with known vertices. [6]

OR

Maximize $Z=6x_1-9x_2$ Subject to

$2x_1-3x_2\le 6;\ x_1+x_2\le 20;\ x_1\ge 0;\ x_2\ge 0$

By using simplex method.

19. Determine using a) Trapezoidal rule b) Simpson's rule, the following integrals. Estimate the error in each case $\int_1^3\frac{dx}{x^2}$, $n=4$.

Estimate the error in using the approximation $n=4$. [6]

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Set B

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group A

1. a) In how many ways can 4 art students and 4 science students be arranged in a round table. [2]

   b) Prove that $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+.............=1-\ln 2$. [2]

   c) If $a*b=e$. Prove that $b=a^{-1}$. [2]

2. a) Find the coordinates of the vertex and focus of the parabola whose equation is $y^2=6y-12x+45$. [2]

   b) Find the ratio in which the line joining the point $(2,4,5)$ and $(3,5,-4)$ is divided by xy plan. [2]

   c) Show that the three points whose position vectors $\overrightarrow{7j}+\overrightarrow{10k}$, $\overrightarrow{-i}+\overrightarrow{6j}+\overrightarrow{6k}$ and $\overrightarrow{-4i}+\overrightarrow{9j}+\overrightarrow{6k}$ from an isosceles triangle. [2]

3. a) Find the derivates of $x^{\cosh^2\frac{x}{a}}$ [2]

   b) Integrate: $\int \sec x \; dx$ [2]

   c) Solve: $e^{x-y}\; dx+e^{y-x}dy=0$ [2]

4. a) If $n=10, \sum x=120 \sum x^2=1530$, find the standard deviation and coefficient of variation. [2]

   b) If dice is thrown 3 times. Getting a 2 or 3 is numbered as a success. Find the probability two successes. [2]

   c) A card is drawn from a well-shuffled deck of 52 cards. What is the probability that is a King or a Diamond. [2]

5. a) From 6 players in how many ways can a selection of 4 be made:

   i. When one particular player is always included.

   ii. When two particular players are always excluded. [4]

   b) Show that the middle term in the expansion of $\left(x-\frac{1}{x}\right)^{2n}$ is $\frac{1.3.5.........(2n-1)}{n!}(-2)^n$ [4]

6. a) Let $*$ be defined on Q^+ by $a*b=\frac{ab}{4}$. Show that $(Q^+, *)$ is an abelian group. [4]

   b) Find the equation of the normal to the parabola $y^2=4ax$ at the point (x_1, y_1) and express in slope form. [4]

7. a) Find the derivative of $\cos(\ln x)$ from first principle. [4]

   OR

   State Rolle's theorem. Interpret it geometrically verify Rolle's theorem for the function $f(x)=x(x-3)^2$ for $x\in[0,3]$

   b) Evaluate: $\int \frac{dx}{1-3\sin x}$. [4]

   OR

   Evaluate: $\int \frac{x}{x^3+1}\, dx$

8. a) Solve: $\frac{dy}{dx}=\frac{y}{x}+\tan\frac{y}{x}$. [4]

   OR

   Solve: $(x^2+1)\frac{dy}{dx}+2xy=4x^2$

b) A certain manufacturing process produces electrical fuses of which 15% are defective. Find the probability that in a sample of 10 fuses selected at random there will be

i. no defective

ii. not more than one defective. [4]

9. Find the direction $Co\sin es\ l,\ m,\ n$ of two lines which are connected by the relations $4l + 3m - 2n = 0$ and $lm - mn + nl = 0$. [6]

OR

Prove that a line which makes angle $x,\ y,\ z,\ t$ with four diagonals of cube is $\cos^2 x + \cos^2 y + \cos^2 z + \cos^2 t = 4/3$.

10. Define vector product of two vectors. Interpret it geometrically. Prove by vector method $Sin(A - B) = SinA.CosB - CosA.SinB$. [6]

11. From the following date. Calculate the coefficient of correlation by Karl Pearson's method

X	6	2	10	4	8
Y	9	11	-	8	7

Arithmetic means of X and Y are 6 and 8 respectively. [6]

Group C

16. a) Determine graphically the solution set of the following system of inequalities $x - y \le 5;\ \ 2x + y \ge 3$ [2]

b) Test the consistency of the given system of equations: [2]
$-2x + y + 3z = 12;\ x + 2y + 5z = 4;\ 6x - 3y - 9z = 24$

c) Evaluate the given integrals using Trapezoidal rule. Give your answer correct upto 3 places of decimals.

$\int_0^1 x^2 dx,\quad n = 2$ [2]

17. a) Rank the given matrix according to their condition [4]

$$\begin{bmatrix} 1 & 8 & -1 \\ -9 & -71 & 11 \\ 1 & 17 & 18 \end{bmatrix}$$

OR

Solve the given system of equation by using Gauss Ellimination method.
method. $x + 2y + 3z = 2;\ x + y - z = 1;\ 2x + 3y + 2z = 3$

b) Apply the method of successive bisection of the root of the equation $x^3 - 4x + 1 = 0$ lying between 1 and 2 correct to two places of decimal by successive bisection method. [4]

18 Minimize $W = 2x + y$ Subject to
$5x + y \ge 9;\ 2x + 2y \ge 10;\ \ x \ge 0;\ \ y \ge 0$ [6]

19. Evaluate an approximate area between the curve $y = (2x + 1)^2,\ x = 1,\ x = 3$ and x-axis taking 4 intervals by Simpson's rule. Also compare it with exact value. [6]

OR

Estimate the integrals $\int_2^3 x dx$ taking $n = 6$ sub intervals by Trapezoidal and Simpson's rule and compare the result with exact value.