FIRST TERM EXAM-2071

Grade: XII Subject: Basic Mathematics F.M.:100

Time: 3:00 hrs. P.M.:40

Set A

Attempt all the questions:

Group-A

[5×3×2 = 30]

1. a. There are 5 boys and 3 girls. In how many ways can they stand in a row so that :
   (i) They may stand anywhere.
   (ii) No two girls are together.

   b. If the coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270, find k.

   c. If $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots\ldots\ldots\ldots$ show that
   $y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\ldots\ldots$

2. a. If the ordinate of a point on the parabola $y^2 = 4ax$ is twice the latus rectum, prove that the abscissa of this point is twice the ordinate.

   b. Find the equation of hyperbola in standard position satisfying the given condition: Focus at (0,5) and vertex at (0,-3).

   c. Find the points on the circle $x^2 + y^2 = 16$ at which the tangents are parallel to the $y - axis$

3 a. Find the derivative of $x^{\cosh\frac{x}{a}}$.

   b. Evaluate: $\lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{\sin x}\right)$ by using L' Hospital's Rule.

   c. If $\vec{a} = (1,2,3)$ and $\vec{b} = (2,3,4)$ find the projection of $\vec{a}$ on $\vec{b}$.

4 a. If $\vec{3i} + \vec{j} - \vec{k}$ and $\vec{\lambda i} - \vec{4j} + \vec{4k}$ are collinear vectors, find the value of λ.

   b. Evaluate: $\int \frac{dx}{\sqrt{1+e^{-2x}}}$

   c. Evaluate: $\int \sqrt{3x^2+5}\ dx$.

5 a. Solve: $e^{x-y}dx + e^{y-x}dy = 0$

   b. Solve: $xdy + (x+1)ydx = 0$

   c. Two dice are thrown together. Find the probability of getting both even digits.

Group-B

[5×2×4=40]

6 a. How many different permutations can be made with letters of the word RANDOM under the following conditions?
   (i) If permutations begin with D,
   (ii) If permutations end with N,
   (iii) If permutations begin with A and end with O,
   (iv) If vowels are never separated.

   b. Find the sum of the infinite series:
   $1 + \frac{1+3}{2!} + \frac{1+3+3^2}{3!} + \frac{1+3+3^2+3^3}{4!} + \ldots\ldots\ldots$

7 a. Find the equation of tangent to the parabola $y^2 = 4ax$ at a point (x_1, y_1) on the parabola.

   b. A tangent to the parabola $y^2 = 8x$ makes an angle of 45^0 with the straight line $y = 3x + 5$. Find its equation.

8 a. Find, from first principle the derivative of $\log\left(\cos\frac{x}{a}\right)$.

   b. Prove that the given vectors are coplanar:
   $\vec{a} - \vec{3b} + \vec{5c}$, $\vec{a} - \vec{2b} + \vec{3c}$, $\vec{-2a} + \vec{3b} - \vec{4c}$ where $\vec{a}$, $\vec{b}$, $\vec{c}$ are the vectors.

9 a. Evaluate: $\int \frac{dx}{2\sin x + 3\cos x}$.

b. Solve: $\left(1+x^2\right)\frac{dy}{dx}+y=e^{\tan^{-1}x}$.

10 a. State and prove theorem of total probability.

b. A bag contains 5 red and 6 white balls. Two balls are drawn at random. Find the probability that

(i) both are white

(ii) first is red and second is white.

Group-C

[5×6 = 30]

11 Prove that the sum of the coefficients of the odd terms in the expansion of $(1+x)^n$ is equal to the sum of the coefficient of the even terms and each is equal to 2^{n-1}. If $(1+x)^n = c_0 + c_1x + c_2x^2 + \dots\dots\dots + c_nx^n$; prove that:

$$c_0c_2 + c_1c_3 + c_2c_4 + \dots\dots\dots + c_{n-2}c_n = \frac{2n!}{(n-2)!(n+2)!}.$$

12 Find the vertices, centre, eccentricity, foci and equations of directrix of the hyperbola

$$9x^2 - 16y^2 - 18x - 64y - 199 = 0$$

13 State Rolle's theorem. Interpret it geometrically. Verify Rolle's theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in $[1,3]$.

14 Define cross product of two vectors and Interpret it geometrically. Prove that $\sin(A-B) = \sin A.\cos B - \cos A.\sin B$.

15 Define degree and order of differential equation.

Solve: $\left(x^3 + y^3\right)dy - x^2ydx = 0$, $f(0) = 1$.

GLOBAL COLLEGE OF MANAGEMENT

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Grade: XII Subject: Basic Mathematics F.M.:100

Time: 3:00 hrs. P.M.:40

Set B

Attempt all the questions:

Group-A
[5×3×2 = 30]

1 a. In how many ways can the letters of the word "LAUGHTER" be arranged so that the vowels may never be separated?

b. If the term independent of x in the expansion of $\left(\sqrt{x}-\frac{k}{x^2}\right)^{10}$ is 405, find k.

c. If $y=\frac{x}{1!}-\frac{x^2}{2!}+\frac{x^3}{3!}-\frac{x^4}{4!}+..........$ show that $x=y+\frac{y^2}{2}+\frac{y^3}{3}+\frac{y^4}{4}+...........$

2 a. Find the equation of the parabola with Focus at $(3,2)$, directrix $x=-4$.

b. Find the equation of an ellipse in standard position satisfying the given condition: Foci$(\pm 2,0)$ and eccentricity $=\frac{1}{2}$.

c. Differentiate: $2\tan^{-1}\left(\tanh\frac{x}{2}\right)$ w.r.t. $'x'$.

3 a. At what angle does the curve $y(1+x)=x$ cut the $x-axis$?

b. Evaluate: $\lim_{x\to 0}\frac{\log\tan x}{\log x}$.

c. If $\vec{a}$ and $\vec{b}$ are two vectors of unit length and θ is the angle between them, show that $\frac{1}{2}\left|\vec{a}-\vec{b}\right|=\sin\theta/2$.

4 a. If $\vec{a}+\vec{b}+\vec{c}=0$, $\left|\vec{a}\right|=3$, $\left|\vec{b}\right|=5$, $\left|\vec{c}\right|=7$, find the angle between $\vec{a}$ and $\vec{b}$.

b. Evaluate $\int\frac{dx}{(11+x)\sqrt{2+x}}$

c. Evaluate $\int\sqrt{x^2-36}\ dx$

5 a. Solve: $\frac{dy}{dx}=e^{x-y}+x^3e^{-y}$

b. Solve: $y(1+xy)dx-xdy=0$

c. If A and B are independent events such that $P(A)=1/2$ and $P(A\cup B)=5/9$ find $P(B)$.

Group-B
[5×2×4=40]

6 a. In how many ways can 5 boys and 5 girls be seated in a row so that
(i) They may sit anywhere
(ii) All girls sit together
(iii) Boys and girls sit alternately
(iv). No two girls sit together?

b. Find the sum of the infinite series;
$1+\frac{1+2}{2!}+\frac{1+2+2^2}{3!}+\frac{1+2+2^2+2^3}{4!}+...........$

7 a. Find the equation of the normal at the point (x_1,y_1) of the parabola $y^2=4ax$

b. A tangent of the parabola $y^2=12x$ makes an angle 45^0 with the straight line $2y=x+3$. Find its equation and the point of contact.

8 a. Find from first principle the derivative of: $\log\left(\sin^{-1} x\right)$.

b. Prove that the following vectors are Coplanar: $-\overrightarrow{a}+\overrightarrow{4b}+\overrightarrow{3c}$, $\overrightarrow{2a}-\overrightarrow{3b}-\overrightarrow{5c}$, $\overrightarrow{2a}+\overrightarrow{7b}-\overrightarrow{3c}$ where $\vec{a}, \vec{b}, \vec{c}$ are any vectors.

9 a. Evaluate: $\int \frac{dx}{1+\sin x+\cos x}$

b. Solve: $\frac{dy}{dx}+y=xy^2$.

10 a. State and prove theorem of compound probability.

b. In a single throw of two dice, find the probability of the following events.

(i) odd number on the first dice and 6 on the second.

(ii) a total of 11.

Group-C

[5×6 = 30]

11 Prove that the sum of the coefficients of the odd terms in the expansion of $(1+x)^n$ is equal to the sum of the coefficient of the even terms and each is equal to 2^{n-1}. If $(1+x)^n = c_0 + c_1x + c_2x^2 + \ldots\ldots\ldots + c_nx^n$; prove that:

$$c_0^{\,2} + c_1^{\,2} + c_2^{\,2} + c_3^{\,3} + \ldots\ldots\ldots + c_n^{\,2} = \frac{(2n)!}{n!n!}.$$

12 Find the vertices, center, eccentricity, foci and equation of directrix of the ellipse: $9x^2+4y^2-18x-16y-11=0$.

13 State Mean value theorem, Interpret it geometrically. Using Mean value theorem, find a point on the curve $y=(x-3)^2$ where the tangent is parallel to the chord joining the points (3,0) and (4,1).

14 Define dot product of two vectors and interpret it geometrically. Prove that $Cos(A+B)=\cos A.\cos B-\sin A.\sin B$.

15 Define degree and order of differential equation.

Solve: $2xydy=\left(y^2-x^2\right)dx;\ f(2)=0$.