GLOBAL COLLEGE OF MANAGEMENT

Second Term Exam – 2070

Grade: XII Subject: Basic Mathematics F.M.: 100
Time: 3:00 hrs. P.M.: 40

Set 'A'

Group A [5×3×2=30]

1. a) Telephone numbers consists of 7 digits and none of them begin with zero. How many telephone numbers could be possible if no digit appears more than once?

   b) If $y = \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$, show that: $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \ldots$

   c) Given an algebraic structure $(Z, *)$, with a binary operation $*$ defined by $m * n = m + n - 1$ for all $m, n \in Z$. Determine the identity element and inverse element.

2. a) Find the eccentricity of the standard ellipse whose latus rectum is the half of the major axis.

   b) The focal distance of a point on the parabola $y^2 = 12x$ is 4. Find the abscissa of this point.

   c) Find the equation of the plane through (3, –4, 5) and parallel to the plane $3x - 4y + 5z = 7$.

3. a) If $\vec{a}.\vec{b} = 48$, $|\vec{a}| = 15$ and $|\vec{b}| = 4$, find $|\vec{a} \times \vec{b}|$.

   b) Find the point on the curve $y = x^3 - 3x^2 + 1$ where the tangents are parallel to the *x*-axis.

   c) Integrate: $\int \frac{dx}{(2x+1)\sqrt{4x+3}}$

4. a) Integrate: $\int \frac{dx}{(\sin x + \cos x)^2}$

   b) Solve: $e^{x-y}\,dx + e^{y-x}\,dy = 0$

   c) Solve: $(x^2 + xy^2)dx + (x^2 y + y^2)dy = 0$

5. a) In a certain distribution, the following results were obtained: mean = 45, median = 48, coefficient of skewness = – 0.4. Find the standard deviation and coefficient of variation.

   b) Determine graphically the solution set of the inequality $3x + 2y \le 12$.

   c) A bag contains 25 tickets, numbered from 1 to 25. A ticket is drawn and then another ticket is drawn without replacement. Find the probability that both tickets will show even numbers.

Group B [5×2×4=40]

6. a) Suppose the prime ministers of 7 South Asian countries meet together to discuss a problem. In how many ways can they sit together at a round table if India and Pakistan prime ministers are not to sit together?

   b) Show that: $1 - \frac{1}{4} + \frac{1.3}{4.8} - \frac{1.3.5}{4.8.12} + \ldots \text{to } \infty = \sqrt{\frac{2}{3}}$

7. a) Sum to infinity the series:
   $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \ldots$

   b) If $a, b \in (G, o)$ then,

   i) $(a o b)^{-1} = b^{-1} o a^{-1}$

   ii) $(a^{-1})^{-1} = a$

8. a) Prove that the latus rectum of a parabola bisects the angle between the tangent and the normal at either extremities of the latus rectum.

   b) Show that the angle between two diagonals of a cube is $\cos^{-1}\left(\frac{1}{3}\right)$ (without using vector method).

9. a) Integrate: $\int \frac{dx}{4+3\sinh x}$

   b) Solve: $\cos^2 x\frac{dy}{dx}+y=1$

10. a) A bag contains 4 white, 5 red and 6 black balls. Two balls are drawn at random. What is the probability that:
   i) both are of the same colours
   ii) both are of different colours

   b) Solve the following system of equations using Gauss elimination method.

$$x+2y+3z=2$$
$$x+y-z=1$$
$$2x+3y+2z=3$$

Group C [6×5=30]

11. Define dot product and interpret it geometrically. Also prove vectorically that: $\cos(A+B)=\cos A\cos B-\sin A\sin B$

12. i) Find by using first principles the derivative of: $\cos^{-1} x$

    ii) Differentiate: $2\tan^{-1}\left(\tanh\frac{x}{2}\right)$ w.r.t. 'x'.

13. From the following data between the ages of husband's and wife's. Calculate the two regression equations and estimate the husband's age when wife's age is 20 and wife's age when husband's age is 30.

Wife's age (X)	18	20	22	23	27	28	30
Husbands age (Y)	23	25	27	30	32	31	35

14. Maximize: $Z=7x+5y$ subject to $x+2y\le 6$; $4x+3y\le 12$; $x,y\ge 0$ by simplex method.

15. Solve the following system of equations by computing inverse matrix using Gauss Jordan method: $x+2y-z=4$; $2x+4y+3z=-7$; $x+3y-6z=15$.

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Set 'B'

Group A [5×3×2=30]

1. a) In how many ways can a conference of 4 boys and 4 girls be seated at a circular table so that no two girls are adjacent?

   b) Prove that: $2\ln x - \ln(x+1) - \ln(x-1) = \frac{1}{x^2} + \frac{1}{2x^4} + \frac{1}{3x^6} + \ldots$

   c) Test for the associativity and commutativity for the operation defined by $a * b = \frac{1}{2}(a+b)$ on Z. $a, b \in Z$.

2. a) Find the equations of the tangent and the normal to the parabola $x^2 = 12y$ at the point whose abscissa is 6.

   b) Find the equation of hyperbola in standard position satisfying the given condition; latus rectum is 4 and eccentricity is 3.

   c) Find the distance between the parallel planes $2x - 2y + z + 1 = 0$ and $4x - 4y + 2z + 3 = 0$.

3. a) If $|\vec{a} \times \vec{b}| = 27$, $|\vec{a}| = 9$ and $|\vec{b}| = 5$, find $\vec{a}.\vec{b}$.

   b) At what angle does the curve $y(1+x) = x$ cut the x-axis?

   c) Integrate: $\int \frac{dx}{4 - 5\sin^2 x}$

4. a) Integrate: $\int \frac{dx}{(11+x)\sqrt{2+x}}$

   b) Solve: $\frac{dy}{dx} = e^{x-y} + x^3.e^{-y}$

   c) Solve: $(1 - \sin x \tan y)dx + \cos x.\sec^2 y\, dy$

5. a) Find the correlation coefficient between the two variables from the following data: n=10, $\sum X=18$, $\sum Y=25$, $\sum X^2 = 90$, $\sum Y^2 = 120$ and $\sum XY=65$.

   b) Determine graphically the solution set of the inequality $y - 2x > -3$.

   c) If A and B are independent events such that $P(A) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{9}$ find P(B).

Group B [5×2×4=40]

6. a) In how many ways can the letters of the word "LOGIC" be arranged so that:
   i) the vowels may occupy odd position?
   ii) no two vowels are together?
   iii) the relative position of the vowels and consonants are not changed?

   b) Show that: $1 + \frac{1}{4} + \frac{1.2}{4.8} + \frac{1.4.7}{4.8.12} + \ldots \text{to } \infty = (2)^{\frac{2}{3}}$

7. a) Sum to infinity the following series:
   $1 + \frac{3}{1!} + \frac{5}{2!} + \frac{7}{3!} + \ldots$

   b) If a and b are the elements of a group (G, o) then, $(x\, o\, a) = b$ have unique solutions in (G, o).

8. a) A tangent to the parabola $y^2 = 8x$ makes an angle of 45^0 with the straight line $y = 3x + 5$. Find its equation.

   b) Find the direction cosines *l*, *m*, *n* of two lines which satisfy the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

9. a) Integrate: $\int \frac{dx}{4 + 3\cosh x}$

   b) Solve: $\left(1 + x^2\right)\frac{dy}{dx} + y = e^{\tan^{-1}x}$

10. a) The chance that A can solve the problem is $\frac{3}{5}$ and the chance that B can solve the problem is $\frac{2}{3}$. Find the probability that:

   i) the problem is solved by A and B
   ii) none of them can solve the problem

   b) Solve the following system of equations using Gauss elimination method.

$$x - y + z = 1$$
$$3x + y + 5z = 11$$
$$4x + 2y + 7z = 16$$

Group C [6×5=30]

11. Define cross product and interpret it geometrically. Also prove vectorically that: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

12. i) Find by using first principles the derivative of: $\sin(\log x)$

   ii) Differentiate: $2\tanh^{-1}\left(\tan\frac{x}{2}\right)$ w.r.t. '*x*'.

13. The equation of two regressions lines are $4X - 5Y + 33 = 0$ and $20X - 9Y = 107$. Find

   i) the mean of X and mean of Y.
   ii) the regression coefficients.
   iii) the correlation coefficients between X and Y.
   iv) the ratio of standard deviations of X and Y.

14. Maximize: $Z = 5x + 3y$ subject to $2x + y \le 40$; $x + 2y \le 50$; $x, y \ge 0$ by simplex method.

15. Solve the following system of equations by computing inverse matrix using Gauss Jordan method: $x + 2y + z = 8$; $2x + 3y + 2z = 14$; $3x + 2y + 2z = 13$.