GLOBAL COLLEGE OF MANAGEMENT

Pre-Board Exam – 2070

Grade: XII | Subject: Mathematics | F.M.: 100

Time: 3 hrs. | P.M.: 35

Set A

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group A

1. a) In how many ways can eight people be seated in a row of eight seats so that two particular persons are always together? [2]

b) If $x = y - \frac{y^2}{2} + \frac{y^3}{3} - \frac{y^4}{4} + \dots$ show that

$y = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots$ [2]

c) If a and b are elements of a group (G, ∗) and a ∗ b = e then show that $b = a^{-1}$. [2]

2. a) Find the equation of the hyperbola the distance between foci is 16 and where eccentricity is $\sqrt{2}$. [2]

b) Find the ratio in which the *yz*-plane divides the join of the points (–2, 4, 7) and (3, –5, 8) and also find the co-ordinates of the point of intersection of this line with the *yz* - plane. [2]

c) If $3\vec{i} + \vec{j} - \vec{k}$ and $\lambda\vec{i} - 4\vec{j} + 4\vec{k}$ are collinear vectors. Find the value of λ. [2]

3. a) Differentiate: $2\tan^{-1}\left(\tan h\frac{x}{a}\right)$ w.r.t. 'x'. [2]

b) Evaluate: $\int \frac{x-2}{\sqrt{2x^2 - 8x + 5}}\, dx$ [2]

c) Solve: $\frac{dy}{dx} = -\frac{1 + \cos 2y}{1 - \cos 2x}$ [2]

4. a) If n = 10, $\Sigma X = 60$, $\Sigma Y = 60$, $\Sigma X^2 = 400$, $\Sigma Y^2 = 580$ and $\Sigma XY = 415$, find the correlation coefficient between the two variables. [2]

b) What is the probability of drawing a heart or a king from a deck of 52 cards? [2]

c) Find the binomial distribution having mean = 12 and variance = 8. [2]

5. a) In how many ways can the letters of the word "COMPUTER" be arranged so that

i. all the vowels are always together?

ii. the vowels may occupy only odd positions? [4]

b) If the three consecutive coefficients in the expansion of $(1 + x)^n$ be 165, 330, 462, find n. [4]

6. a) Define group. Let G = {1, – 1, i, – i} where i is an imaginary unit and * stands for the binary operation of multiplication. Show that (G, *) forms a group. [4]

b) Find the condition of tangency of the straight line $y = mx + c$ to the parabola $y^2 = 4ax$. [4]

7. a) Find from first principles the derivative of: $\tan^{-1}x$ [4]

'OR'

State Rolle's theorem. Apply Rolle's theorem to the function $f(x) = \sqrt{1 - x^2}$ on [–1, 1]. Observe that f fails to be differentiate at the end points of the interval.

b) Evaluate: $\int \frac{\text{Sin}x\, dx}{4\tan x - \text{cosec}x\,\text{Sec}x}$ [4]

'OR'

Evaluate: $\int \frac{\cos x - \sin x}{\sqrt{\sin 2x}}\, dx$

8. a) Solve: $\frac{dy}{dx} = \frac{y}{x} + \sin\frac{y}{x}$ [4]

'OR'

Solve: $\cos^2 x \frac{dy}{dx} + y = 1$

b) If 20% of the electric bulbs manufactured by a company are defective, find the probability that out of 4 bulbs chosen at random a) 1 b) 0 c) at most 2 bulbs will be defective. [4]

9. Find the angle between the lines whose direction cosines l, m, n satisfies the equation. $3l + m + 5n = 0$, $6mn - 2nl + 5lm = 0$ [6]

'OR'

Define direction cosines of a line. Prove that the lines whose direction cosines are given by the relation $al + bm + cn = 0$ and $fmn + gnl + hlm$ are perpendicular if $\frac{f}{a} + \frac{g}{b} + \frac{h}{c} = 0$.

10. Prove vectorically that: [6]

a) b = c cosA + a cosC

b) $a^2 = b^2 + c^2 - 2bc \cos A$.

11. The equation of two regressions lines are

$4X - 5Y + 33 = 0$ and $20X - 9Y = 107$

Find: a) the mean of X and mean of Y.

b) the regression coefficients

c) the correlation coefficient between X and Y

d) the ratio of standard deviations of X and Y. [6]

Group 'C'

16. a) Convert the hexadecimal numeral AFB2 to binary form. [2]

b) Find the condition number for the following system of equation? $\begin{bmatrix} 1 & 2 \\ 2 & 3.999 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$

What can you say about the system? [2]

c) Determine graphically the solution set of

$2x + y \geq 2,\ x \geq 0,\ y \geq 0$. [2]

17. a) Solve the given system of equations by Gauss elimination method: [4]

$2x + 3y + 4z = 20$

$3x + 4y + 5z = 26$

$3x + 5y + 6z = 31$

'OR'

Use the Gauss-Seidel method to solve the system.

$3x + y = 6$

$x - 3y = 5$

b) Find an approximation to $\sqrt[3]{25}$ correct to within 10^{-1} using the Bisection Method. [4]

18. A person requires minimum 10, 12 and 12 units of chemicals A, B and C respectively for his garden. A liquid product contains 5, 2 and 1 units of A, B and C respectively per jar. A dry product contains 1, 2 and 4 units of A, B and C per carton. If the liquid product sells for Rs. 3 per jar and dry product sells for Rs. 2 per carton, how many of each should be purchased in order to minimize the cost and meet the requirements. Formulate this problem and solve graphically. [6]

'OR'

Using Simplex method, Minimize $W = 3x + 2y$

Subject to

$2x + y \geq 6$

$x + y \geq 4$

$x \geq 0,\ y \geq 0$

19. Determine using (a) trapezoidal rule (b) Simpson's rule, the following integrals, estimate the error in $\int_0^4 x^2\,dx$, n = 4. [6]

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Set B

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group A

1. a) In how many arrangements can be made from the letters of the word "MINIMUM" so that the three M's do not come together? [2]

   b) If the coefficient of x in the expansion of $\left(x^2+\frac{k}{x}\right)^5$ is 270, find k. [2]

   c) Define $a*b=a^b$ on Z. Show that '*' is not associative binary operation. [2]

2. a) Find the equation of the ellipse whose foci are $(\pm 2, 0)$ and the length of the latus rectum is 6. [2]

   b) If α, β and γ are the direction angles of a line. Prove that: $\cos 2\alpha+\cos 2\beta+\cos 2\gamma+1=0$. [2]

   c) ABCD is a parallelogram, G is the point of intersection of its diagonals and if O is any point show that: [2]
   $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}=4\overrightarrow{OG}$

3. a) Using L-Hospital's rule, evaluate: $\lim_{x\to 0}\frac{\tan x-x}{x-\sin x}$ [2]

   b) Integrate: $\int\frac{dx}{e^x+e^{-x}}$ [2]

   c) Solve: $(1-\sin x\tan y)dx+\cos x.\sec^2 y\,dy=0$ [2]

4. a) In a distribution, the difference of the two quartiles is 20 and their sum is 70 and the median is 36. Find the coefficient of skewness. [2]

   b) What is the probability that there will be 5 Sundays in the month of July? [2]

   c) Find the focus and the equation of directrix of the parabola: $x^2=16y$ [2]

5. a) In how many ways can the letters of the word "MONDAY" be arranged? How many of these arrangements do not begin with M? How many begin with M and do not end with Y? [4]

   b) Sum to infinity of given series:
   $1+\frac{3}{1!}+\frac{5}{2!}+\frac{7}{3!}+\dots$ [4]

6. a) Define group. Given the algebraic structure (G, *) with G = {1, w, w^2} where w represents an imaginary cube root of unity and * stands for the binary operation of multiplication, show that (G, *) is a group. [4]

   b) Show that the angle between the tangents to the parabola $y^2 = 4x$ and $x^2 = 4y$ at their points of intersection other than the origin is $\tan^{-1}\frac{3}{4}$. [4]

   'OR'

   Find the vertices, centre, eccentricity, foci of the hyperbola.
   $9x^2-16y^2-18x-64y-199=0$

7. a) Integrate: $\int\frac{1-x}{x^2+x^3}\,dx$ [4]

   b) Solve: $(1-x^2)\frac{dy}{dx}-xy=1$ [4]

8. a) From the following table calculate the correlation coefficient by Karl Pearson's method.

X:	10	12	20	?	16	14
Y:	9	12	15	18	14	16

Arithmetic mean of X is 15. [4]

'OR'

Find the regression equations of y on x from the following data:

X:	5	9	13	17	21
Y:	3	8	13	18	23

Estimate the value of y when $x = 12$.

b) The incidence of occupation disease in an industry is such that the workmen have a 20% chance of suffering from it, what is the probability that out of six workmen four or more will contract the disease? [4]

9. Show that the straight lines whose direction cosines are given by the equation $al + bm + cm = 0$ and $ul^2 + vm^2 + wn^2 = 0$ are perpendicular if $a^2(v+w) + b^2(u+w) + c^2(u+v) = 0$. [6]

10. Define vector product of two vectors. Interpret it geometrically. Prove by vector method: $\frac{Sin\,A}{a} = \frac{Sin\,B}{b} = \frac{Sin\,C}{c}$ [6]

11. State Rolle's Theorem. Interpret it geometrically. Verify Rolle's theorem for the function: [6]

$f(x) = x(x-3)^2$ for $x \in [0, 3]$

'OR'

Find from first principle the derivative of: $\ln \tan^{-1} x$

Group C

16. a) Given the velocity of objects as follows:

Time(s)	0	1	2	3
Velocity:	0	10	12	14

Obtain an estimate of the distance travel in the interval [0, 3] by trapezoidal rule. [2]

b) Convert the given decimal numerals 41.6875_{10} to binary form. [2]

c) Determine graphically the solution set of $x - 5y \le 5,\ x \ge 0,\ y \ge 0$. [2]

17. a) Use the Gauss-Seidal method to solve the system: [4]

$3x + y - z = 2;\ 2x - 5y + z = 20;\ x - 3y - 8z = 3$

'OR'

Use the Gauss-elimination method to solve the system:

$2x - 3y + 3z = 27;\ 4x + y - 2z = 0;\ -6x - 4y + 2z = 0$

b) How many partition points must be considered to have the approximated value of $\int_1^2 \frac{2}{x}dx$ within the accuracy of 10^{-4}?

Use Simson's $\frac{1}{3}$ rule. [4]

18. By using Simplex method, maximize $p = 100x + 10y$ subject to $2x + 5y \le 20;\ 2x + y \le 12;\ x,\ y \ge 0$. [6]

19. Use Newton-Raphson method, find a positive root of $x^3 - 2x - 5 = 0$ lying between 2 and 3 correct to three places of decimals. [6]

'OR'

Use the method of successive bisection to find the root of the equation $x^3 - 2x - 5 = 0$ in (2, 3) correct to three places of decimal.

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