GLOBAL COLLEGE OF MANAGEMENT

First Term Exam - 2070

Grade: XII　　Subject: Mathematics　　F.M.: 100

Time: 3:00 hrs.　　P.M.: 40

Set-B

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions:

Group 'A' [5×3×2=30]

1. a) There are 5 boys and 3 girls. In how many ways can they stand in a row so that no two girls are together?

   b) Find the middle terms in the expansion of $\left(\frac{x}{a}-\frac{a}{x}\right)^{2n+1}$.

   c) Show that: $\frac{2}{3!}+\frac{4}{5!}+\frac{6}{7!}+..\infty=\frac{1}{e}$

2. a) Find the equation of the parabola with vertex at (–1, 1) and directix $y = 3$.

   b) Deduce the equation to the hyperbola in the standard form with a focus at (–7, 0) and eccentricity $\frac{7}{4}$.

   c) If the co-ordinates of P, Q, R and S be (1, 2, 2), (2, 4, 0), (–3, 0, 1) and (–1, –2, 2) respectively, find the projection of RS on PQ.

3. a) Find the ratio in which the *yz*-plane divides the line joining the points (4, 6, 7) and (–1, 2, 5).

   b) If $\vec{a}+\vec{b}+\vec{c}=0$, $|\vec{a}|=3, |\vec{b}|=5$, $|\vec{c}|=7$, find the angle between $\vec{a}$ and $\vec{b}$.

   c) If the points with position vectors $\vec{i}+2\vec{j}+\vec{k}$, $2\vec{i}-\vec{j}+3\vec{k}$ and $5\vec{i}-10\vec{j}+\lambda\vec{k}$ are collinear, find the value of λ.

4. a) Find the value of $\lim_{x\to 0}\frac{\log\tan x}{\log x}$

   b) Prove that: $\int \sec x dx = \log\left|\tan\left(\pi/4 + x/2\right)\right| + C$

   c) Integrate: $\int\sqrt{2ax-x^2}\ dx$

5. a) Solve: $\sec^2 x\tan y\,dx+\sec^2 y\tan x\,dy=0$

   b) Solve: $(x^2-ay)dx-(ax-y^2)dy=0$

   c) Find the probability of getting two heads twice in 5 tosses of two coins.

Group 'B' [5×2×4=40]

6. a) In how many ways can the letters of the word "NEPAL" be arranged so that

      i) all the vowels are always together?

      ii) the vowels may occupy only odd positions?

      iii) the relative positions of vowels and consonants are not changed?

   b) If the three consecutive coefficients in the expansion of $(1+x)^n$ is 165, 330, 462, find n.

7. a) Find the equation of the tangent at the point (x_1, y_1) of the parabola $y^2=4ax$

   b) Find the equation of the plane through the points (–1, 1, 1) and (1, –1, 1) and perpendicular to the plane $x + 2y + 2z = 5$.

8. a) Examine whether the given vectors are linearly dependent or independent: $\vec{2i}+\vec{3j}+\vec{4k}, \vec{i}-\vec{j}+\vec{2k}$ and $\vec{5i}+\vec{6j}+\vec{8k}$

   b) Find, from first principles, the derivative of $\log\left(\cos\frac{x}{a}\right)$

9. a) Evaluate: $\int\frac{dx}{3+4\cosh x}$

   b) Solve: $xdy-ydx=\sqrt{x^2+y^2}\ dx$

10. a) State and prove theorem of compound probability

b) The letters of the word “TRIANGLE “are arranged at random. Find the probability that the word so formed

(i) starts with T. (ii) ends with E.

Group ‘ C’ [6×5=30]

11. Prove that the sum of all binomial coefficients involved in a binomial expansion of $(1+x)^n$ is 2^n.

If $(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots\ldots + C_nx^n$ then, prove that

$$C_0C_1 + C_1C_2 + C_2C_3 + \ldots\ldots\ldots + C_{n-1}C_n = \frac{2n!}{(n+1)!\,(n-1)!}$$

12. Find the angle between two lines whose direction cosines are (l_1, m_1, n_1) and (l_2, m_2, n_2). Also deduce the condition of perpendicularity and parallelism.

13. Define dot product and interpret it geometrically. Also prove vectorically that $Cos(A-B) = CosACosB + SinA.SinB$

14. Find the vertices, centre, focus, eccentricity and the equation of directrix of ellipse: $9x^2 + 4y^2 - 18x - 16y - 11 = 0$

15. State Mean value theorem and interpret it geometrically. Using mean value theorem, find a point on the parabola $y = (x-3)^2$ where the tangent is parallel to the chord joining the points (3,0) and (4,1).

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Set-A

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt all the questions:

Group 'A' [5×3×2=30]

1. a) In how many ways can the letters of the word LAUGHTER be arranged so that the vowels may never be separated.

   b) Find the coefficients of x^6 in the expansion of $\left(3x^2 - \frac{1}{3x}\right)^9$

   c) Prove that: $\frac{1}{1.2} + \frac{1}{2.2^2} + \frac{1}{3.2^3} + \ldots\ldots = \log_e 2$.

2. a) Find the equation of the parabola with Vertex at (-5,-3), and end of the latus rectum (-1,5) and (–1, –1).

   b) Determine the equation of the hyperbola with a focus at (6,0) and a vertex at (4,0)

   c) If α, β and γ are the direction angles of a line , prove that $\cos 2\alpha + co2\beta + co2\gamma + 1 = 0$.

3. a) Find the ratio in which the line joining the points (-2,4,7) and (3,-5,-8) is divided by the xy – plane.

   b) If θ be the angle between two unit vectors $\vec{a}$ and $\vec{b}$. Show that $\frac{1}{2}\left|\vec{a} - \vec{b}\right| = \sin\frac{\theta}{2}$

   c) Find the area of the triangle determined by the vectors $-\vec{2i} + \vec{3j} - \vec{k}$ and $\vec{3i} - \vec{4j} + \vec{2k}$.

4. a) Evaluate: $\lim_{x\to 0} \frac{xe^x - \log(x+1)}{x^2}$

   b) Prove that : $\int \cos ecx dx = \log\left|\tan\frac{x}{2}\right| + c$

   c) Integrate: $\int \frac{dx}{\sqrt{2ax - x^2}}$

5 a) Solve: $\frac{dy}{dx} = \frac{xy + y}{xy + x}$

   b) Solve: $ydx - xdy = xydy$

   c) In a single throw of two dice, find the probability of the odd number on the first die and 6 on the second.

Group 'B' [5×2×4=40]

6. a) How many words can be formed from the letters of the Word "ENGLISH" ? How many of these do not begin with E? How many of these begin with E and do not end with H?

   b) If the coefficients in three successive terms of the expansion of $(1+x)^n$ are 220, 495 and 792, find n .

7. a) Find the equation of the normal at the point (x_1, y_1) of the parabola $y^2 = 4ax$ in standard form.

   b) Find the direction cosines l, m, n of two lines which satisfy the equations: $l + m + n = 0$ and $2lm - mn + 2nl = 0$

8 a) Prove that the given vectors are coplanar: $\vec{a} - \vec{3b} + \vec{5c}$, $\vec{a} - \vec{2b} + \vec{3c}$, $-\vec{2a} + \vec{3b} - \vec{4c}$

   b) Find, from first principles, the derivative of $\log\left(\cos^{-1} x\right)$.

9 a) Evaluate: $\int \frac{dx}{2\sin x + 3\cos x}$

   b) Solve: $\frac{dy}{dx} + \frac{2x}{(1+x^2)}y = \frac{1}{(1+x^2)^2}$.

10. a) State and prove theorem of total probability.

    b) Three balls are drawn in succession from a bag containing 8 white and 6 black balls. What is the probability that:

    i) all three balls are white.

    ii) one is white and 2 is black.

Group ' C' [6×5=30]

11. If $(1+x)^n = C_0 + C_1 x + C_2 x^2 + \ldots\ldots\ldots + C_n x^n$ prove that:

$$C_0C_2 + C_1C_3 + C_2C_4 + \ldots\ldots\ldots + C_{n-2}C_n = \frac{2n!}{(n-2)!\,(n+2)!}$$. Also

find the middle term in the expansion of: $\left(x + \frac{1}{x}\right)^{18}$.

12. Find the center, vertex, eccentricity, the foci and equation of directrix of the given ellipse. $x^2 + 4y^2 - 4x + 24y + 24 = 0$.

13. Prove that the general equation of the first degree in x, y, z represent a plane. Also find the equation of the plane through (1,2,3) and parallel to the plane $3x - 4y + 5z = 0$

14 Define cross product and interpret it geometrically. Also prove vectorically that: *Sin(A-B) = SinA.CosB-CosA.SinB.*

15. State Rolle's theorem and interpret it geometrically. Verify Rolle's theorem for *f(x)=Sin*2*x* in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.