GLOBAL COLLEGE OF MANAGEMENT

Second Term-2072

Grade: XI **Subject: Basic Mathematics** **F.M.:100**

Time: 3 Hrs. **P.M.: 40**

Set A

Group A **[5×3×2=30]**

Attempt all the questions

1. a. Rewrite using modulus sign for $-3 \le x+1 \le 2$.

   b. Construct the truth table of the compound statement $p \Rightarrow \sim q$.

   c. Let $A=\{1, 2, 3, 4, 5\}$ and $B=\{2, 4, 6, 8\}$. Find the relation R from set A to set B defined by $y = 2x$. Is this relation a function?

2. a. Without expanding the determinant, prove that $\begin{vmatrix} 1/a & a^2 & bc \\ 1/b & b^2 & ca \\ 1/c & c^2 & ab \end{vmatrix} = 0$

   b. Find the inverse of the matrix $\begin{pmatrix} 2 & -1 \\ -3 & 3 \end{pmatrix}$.

   c. Solve by using Cramer's rule $\frac{x}{4}+\frac{y}{10}=1$ and $\frac{x}{5}+\frac{3y}{25}=1$.

3. a. Show that the roots of $ax^2+bx+c=0$ are rational if $a+b+c=0$ and a, b, c are rational.

   b. If $\propto, \beta$ are roots of $ax^2+bx+c=0$, find the quadratic equation whose roots are $\propto \beta^{-1}$ and $\beta \propto^{-1}$.

   c. Express the given complex number in the polar form: $-3-\sqrt{3}\, i$.

4. a. Evaluate: $\lim_{x \to b} \frac{\sqrt{2x}-\sqrt{3x-b}}{\sqrt{x}-\sqrt{b}}$

   b. Evaluate: $\lim_{x \to y} \frac{\tan y - \tan x}{y-x}$

   c. Find $\frac{dy}{dx}$ when $x = t \ln t, \; y = \frac{\ln t}{t}$

5. a. Find the intervals on which the given function is increasing or decreasing: $f(x) = x^2 + 4x - 5$

   b. Integrate: $\int \cos^2(ax).dx$

   c. Integrate: $\int \sec x.dx$

Group B **[5×2×4=40]**

6. a. Let $A, B, and\ C$ be the subsets of universal set $\cup$. Then prove that $A-(B \cap C)=(A-B)\cup(A-C)$. Also define complement of a set with an example.

   b. State De-Moivre's theorem and use it to solve $z^2 = -1$.

7. a. Prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = (a-b)(b-c)(c-a)(ab+bc+ca)$

   b. Solve the system of linear equations by Row –equivalent method $5x+2y+z=1, \quad 3x+y-z=2, \quad x+y+2z=-1$.

8. a. State principle of Mathematical Induction and use it to prove that $x^n - y^n$ is divisible by $x - y, \quad n \in N$

b. Define removable discontinuity of a function $f(x)$ at a point $x = a$. Discuss the continuity or discontinuity of function at given point

$$f(x) = \begin{cases} 3 + 2x & \text{for } \frac{-3}{2} \le x < 0 \\ 3 - 2x & \text{for } 0 \le x < 3/2 \\ -3 - 2x & \text{for } x >, 3/2 \end{cases} \text{ at } x = 3/2$$

9. a. Find the derivative of e^{ax+b} from the first principle.

b. A ladder *25m* long is leaning against a wall. The bottom of the ladder is pulled along the ground, away from the wall at the rate of *4m/s*. How fast its height on the wall is decreasing when the foot of the ladder is *7 m* away from the wall.

10. a. Integrate: $\int \frac{1}{x^2\sqrt{x^2 - a^2}}\, dx$

b. Integrate: $\int \sin^4 x.\ cox^3 x.\ dx$

Group C **[5×6=30]**

11. Define bijective function. Determine whether the given function is one-to one or onto or both or neither $f : R \to R$ defined by $f(x) = 3x - 7$.

12. Derive the conditions for the equations $ax^2 + bx + c = 0$ and $a_1x^2 + b_1x + c_1 = 0$ may have one common root and use it to find the value of p for the equation $3x^2 - 2x + p = 0$ and $6x^2 - 17x + 12 = 0$.

13 Define derivative of the function at a point. Interpret the derivative of a function Geometrically.

14. Define absolute value of a complex number. Find the cube roots of unity and discuss its properties.

15. What are criteria for the function $f(x)$ to have maximum or minimum? Find the maxima or minima and point of inflection of $y = 2x^3 - 9x^2 + 12x - 4$.

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Set B

Group A **[5×3×2=30]**

Attempt all the questions

1. a. Prove that for any two sets: $A \subseteq B \Rightarrow \overline{B} \subseteq \overline{A}$.

   b. Construct a truth table of the compound statement $p \wedge \sim q$.

   c. Let $A = \{1, 2, 3\}$, find the relation on A satisfying $x + y < 4$ and also find its range. Is this relation a function ?

2. a. Without expanding the determinant prove that
$$\begin{vmatrix} 1 & bc & bc\,(b+c) \\ 1 & ca & ca\,(c+a) \\ 1 & ab & ab\,(a+b) \end{vmatrix} = 0$$

   b. Let $A = \begin{pmatrix} 2 & 0 \\ 1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 1 \\ 3 & 2 \end{pmatrix}$. Show that $(AB)^T = B^T.A^T$

   c. Solve by using Cramer's rule: $\frac{x}{2} - \frac{2y}{5} = \frac{-23}{60}, \quad \frac{2x}{3} + \frac{y}{4} = -\frac{1}{4}$

3. a. If the roots of $(c^2 + d^2)x^2 - 2(ac + bd)x + (a^2 + b^2) = 0$ are equal then prove that $\frac{a}{b} = \frac{c}{d}$.

   b. If $\propto, \beta$ are roots of $ax^2 + bx + c = 0$, find the equation whose roots are $\propto \beta^{-1}$ and $\beta \propto^{-1}$.

   c. If w is a complex cube root of unity, find the value of $(1 + w - w^2)^5 + (1 - w + w^2)^5$

4. a. Evaluate: $\lim_{x \to 1} \frac{x - \sqrt{2 - x^2}}{2x - \sqrt{2 + 2x^2}}$

   b. Evaluate: $\lim_{h \to 0} \frac{\tan(x + h) - \tan x}{h}$

   c. Find $\frac{dy}{dx}$ when $x = e^t \cos t, \; y = e^t \sin t$

5. a. Find the intervals on which the given function is increasing or Decreasing: $f(x) = x^2 - 2x + 10$.

   b. Integrate: $\int \sin^2(ax).\, dx$

   c. Integrate: $\int \cos ecx \,.\, dx$

Group B **[5×2×4=40]**

6. a. Let A, B and C be the subset of universal set U. Then prove that $A - (B \cup C) = (A - B) \cap (A - C)$. Also define difference of two sets with example.

   b. State De-Moivre's theorem and use it to solve $z^2 = i$

7. a. Prove that

$$\begin{vmatrix} 1 & 1 & 1 \\ x^2 & y^2 & z^2 \\ x^3 & y^3 & z^3 \end{vmatrix} = (y-z)(z-x)(x-y)(yx+zx+xy)$$

b. Solve the system of linear equations by Row-equivalent method:
$3x-2y+5z=28,\ -2x+3y+z=30,\ x+y-2z=2$

8. a. State principle of Mathematical Induction and use it to prove
$1.2+2.3+\text{..........}+n(n+1)=\dfrac{n(n+1)(n+2)}{3}, n\in N$.

b. Define removable discontinuity of a functions $f(x)$ of a point $x=a$. A function $f(x)$ is defined as

$$f(x)=\begin{cases} 5x^2+3 & \text{for } x>1 \\ 9 & \text{for } x=1 \\ 6x+2 & \text{for } x<1 \end{cases}$$

. Show that $f(x)$ has removable discontinuity at $x=1$.

9. a. Find the derivative of: $\ln(ax+b)$ from the first principle.

b. Water is running into a conical reservoir, *10 cm* deep and *5 cm* in radius at the rate of *1.5* cm^3/min. At what rate is the water level rising when the water is *4cm* deep.

10. a. Integrate: $\int \frac{1}{x^2\sqrt{a^2-x^2}}\, dx$

b. Integrate: $\int \tan^2 x.\ \sec^4 x.\ dx$

Group C [5×6=30]

11. Define bijective function. Determine whether the given function is one to-one or onto or both or neither. $f: R \to R$ defined by $f(x)=2x+5$.

12. Derive the condition for the quadratic equation $ax^2+bx+c=0$ and $a_1x^2+b_1x+c_1=0$ may have one root common and use it to find the value of m for the equations $2x^2-7x+3=0,\ 3x^2-mx-3=0$.

13. Define derivative of a function at a point. Interpret the derivative of the function geometrically.

14. Define modulus of a complex number. Find the cube roots of unity and discuss its properties.

15. What are the Criteria for the function $f(x)$ to have maximum and minimum? Find maxima and minima and point of inflection of $y=4x^3-6x^2-9x+1$