GLOBAL COLLEGE OF MANAGEMENT

FIRST TERM EXAM – 2072

Grade: XI **Subject: Mathematics** **F.M.:100**
Time: 3:00 hrs. **P.M.:40**

Set-A

Group 'A' [5×3×2=30]

1. a) What is power set of a given set. Write the power set of $\{a, b\}$.
   b) For any two sets, prove that $A - B \subseteq \overline{B}$
   c) let A= [-5,3) and B= [-3, 5]. Find $A \cap B$ and A-B.

2. a) Define disjunction of two statements construct a truth table of $p \vee \sim q$
   b) Construct a truth table of $p \Rightarrow \sim q$.
   c) Rewrite by using modulus sign $-7 \le x + 2 \le -1$.

3. a) If $a - ib = \sqrt{\frac{1-i}{i-1}}$, prove that $a^2 + b^2 = 1$.
   b) If w be complex cube roots of unity. Then evaluate: $(1-w)(1-w^2)(1-w^4)(1-w^8)$
   c) Express the given complex number in the polar form: $\sqrt{3} - i$

4. a) For what value of 'a' will the equation $x^2 - (3a-1)x + 2(a^2-1) = 0$ have equal roots?
   b) Form a quadratic equation whose roots are the squares of the roots of $3x^2 - 5x - 2 = 0$
   c) State principle of mathematical induction. If $P(n) = 4^n - 1$ is divisible by 3, examine $P(2)$.

5. a) If $f(x) = \frac{x^2-4}{|x-2|}$. Does $\lim_{x \to 2} f(x)$ exist ?
   b) Evaluate : $\lim_{x \to \infty} \sqrt{x}(\sqrt{x} - \sqrt{x-a})$.
   c) Evaluate: $\lim_{x \to y} \frac{\sin x - \sin y}{x - y}$

Group 'B' [5×2×4=40]

6. a) Let A, B and C be the subset of a universal set U. then prove that :
   (i) $\overline{A \cap B} = \overline{A} \cup \overline{B}$
   (ii) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
   b) Solve the given inequality: $x^3 + x^2 - 6x \ge 0$

7. a) Define tautology write a truth table of $\sim (p \vee q) \Leftrightarrow [(\sim p) \wedge (\sim q)]$. Hence draw a conclusion from the truth table.
   b) It the equations $x^2 + ax + bc = 0$ *and* $x^2 + bx + ca = 0$ have a common root, prove that their other roots will satisfy the equation $x^2 + cx + ab = 0$.

8. a) Using De-moivre's theorem , solve $z^3 = -1$.
   b) Prove by the principle of mathematical induction for all $\in N$: $1^2+2^2+3^2+\ldots\ldots\ldots+n^2 = \frac{1}{6}$ n(n+1) (2n+1)

9. a) Evaluate : $\lim_{y \to x} \frac{y \tan y - x \tan x}{y - x}$
   b) Discuss the continuity or discontinuity of the given function $f(x) = \begin{cases} x^2 - 5 & for \quad x < 4 \\ 8 & for \quad x = 4 \\ 2x + 3 & for \quad x > 4 \end{cases}$ *at* $x = 4$

10. a) Find from first principle the derivative of $\frac{1}{\sqrt{3x+4}}$.

b) Find the derivative of

(i) $\sqrt{x+\sqrt{x}}$

(ii) $(1-3x)^2\,(2+3x^2)^3$

Group 'C' [6×5=30]

11. Derive the condition for the quadratic equations $ax^2+bx+c=0$ and $a_1x^2+b_1x+c_1=0$ may have one root common and two roots common.

12. Define conjugate of a complex number. For any two complex number Z and W, prove that $|z+w|\le|z|+|w|$.

13. For all rational values of n, prove that

$$\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$$

14. Define removable discontinuity at x=a. Find from first principle the derivative of $\sqrt{\cos 2x}$

15. Find $\frac{dy}{dx}$:

(i) $x^2y^2=x^2+y^2$

(ii) $x=3at^2,\ y=3at^3$

(iii) $y=\cot^4(\sqrt{\sec(1-3x^2)})$

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Grade: XI Subject: Mathematics F.M.:100
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Set-B

Group 'A' [5×3×2=30]

1. a) Define symmetric difference of two sets. Write the power set of {a, b}
   b) For any two sets, prove that $B - \overline{A} = A \cap B$.
   c) Let A= [-3, 2) and B= [-2, 3), find A∩B and A-B

2. a) Define conjunction of two statements. Construct a truth table of $\sim p \wedge q$.
   b) Construct a truth table of $\sim p \Rightarrow q$
   c) Rewrite by using the modulus sign $-5 \le x-1 \le -2$

3. a) If $x - iy = \sqrt{\frac{1-i}{i-1}}$, $prove that\, x^2 + y^2 = 1$.
   b) If w be a complex cube roots of unity. Then evaluate $(1-w+w^2)^4 + (1+w-w^2)^4$
   c) Express the given complex number in the polar form, $1-\sqrt{3}i$

4. a) For what value of m will be the equation $(m+2)x^2 - 2(m+4)x + (m+7) = 0$ have equal roots.
   b) Form a quadratic equation whose roots are the squares of the roots of $3x^2 - 4x + 1 = 0$
   c) State principle of mathematical induction. If P (n) = $9^n - 1$ is divisible by 4, examine P (3).

5. a) If $f(x) = \frac{x^2-1}{|x-1|}$ .Does $\lim_{x \to 1} f(x)$ exists?
   b) Evaluate : $\lim_{x \to 4} \frac{x^2-16}{\sqrt{3x+4} - 4}$
   c) Evaluate : $\lim_{x \to 0} \frac{\cos(ax) - \cos(bx)}{x^2} = \frac{1}{2}(b^2 - a^2)$.

Group 'B' [5×2×4=40]

6. a) let A, B and C be the subsets of a universal set U. Then prove that:
   (i) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
   (ii) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
   b) let *x* be any real number and 'a'be any positive real number, then prove that $|x| < a \Leftrightarrow -a < x < a$.

7. a) Define contradiction. Write a truth table of quadratic $\sim (p \Rightarrow q) \Leftrightarrow p \wedge (\sim q)$,
   Hence draw a conclusion from the truth table.
   b) If the quadric equation $x^2 + px + q = 0$ *and* $x^2 + qx + p = 0$ have a common root, prove that either $p=q$ or $p+q+1=0$

8. a) State and prove De- Moivre's theorem.
   b) Prove by the principle of mathematical induction for all $n \in N$: $1.3+2.4+3.5+ \ldots\ldots\ldots +n(n+2) = \frac{n(n+1)(2n+7)}{6}$

9. a) Evaluate: $\lim_{y \to x} \frac{y \cot y - x \cot x}{y - x}$

b) Discuss the continuity or discontinuity of the given function

$$f(x) = \begin{cases} 5x^2 + 3 & \text{for } x > 1 \\ 9 & \text{for } x = 1 \\ 6x + 2 & \text{for } x < 1 \end{cases} \text{ at } x = 1.$$

10. a) Find from first principle the derivative of $\frac{1}{\sqrt{2x+3}}$

b) Find the derivative of

(i) $\frac{1+\sqrt{x}}{1-\sqrt{x}}$

(ii) $(x-1)\sqrt{x^2 - 2x + 2}$

Group 'C' **[6×5=30]**

11. Prove that the quadratic equations $ax^2+bx+c=0$ $(a \neq 0)$ has two and only two roots.

12. Define modulus of a complex number. For any two complex number z and w, prove that $|z - w| \leq |z| + |w|$

13. Prove geometrically: $\lim_{\theta \to 0} \frac{\sin \theta}{\theta} = 1$, where θ is measured in radian.

Use it to fine the limit of : $\lim_{\theta \to 0} \frac{\sin^2 2\theta}{\theta}$

14. Define removable discontinuity at $x = a$. find from first principle the derivative of $\sqrt{\sin 2x}$.

15. Find $\frac{dy}{dx}$ if

i) $x^3 + y^3 = x^3 y^3$

ii) $x = ct^2, y = \frac{c}{t^2}$

iii) $y = \tan^3(\sqrt{\operatorname{cosec} cx^3})$

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