



Grade: XI
Time: 3:00 hrs.

Subject: Basic Mathematics

F.M.:100
P.M.:40

Set A

Group A [5×3×2=30]

1.
 - a. If p and q are two statements then prove that $(p \vee q) \vee (\sim p)$ is a tautology.
 - b. Find the domain of the function $y = \frac{1}{\sqrt{3x-2}}$.
 - c. Prove that $\overline{A \cup B} = \bar{A} \cap \bar{B}$.

2.
 - a. Evaluate: $\lim_{x \rightarrow 0} \frac{1 - \cos px}{1 - \cos qx}$.
 - b. A function $f(x)$ is defined as

$$f(x) = \begin{cases} x^2 - 1 & \text{When } x < 1 \\ x^2 + 1 & \text{When } x \geq 1 \end{cases}$$
 Examine whether the function is continuous or not at $x = 1$.
 - c. Find the derivative of $y = \log(\sec x + \tan x)$

3.
 - a. A circular plate of metal expands by heat so that its radius increasing at the rate of 0.25cm/sec. Find the rate at which the surface area is increasing when the radius is 7 cms.
 - b. Evaluate: $\int \sin^3 x \, dx$
 - c. Evaluate: $\int \frac{1}{e^x - 1} \, dx$

4.
 - a. For what value of k will the equation $(k+1)x^2 + 2(k+3)x + 2k + 3 = 0$ have equal roots.
 - b. Express the complex number $-1 + \sqrt{3}i$ into polar form.
 - c. Define symmetric and skew symmetric matrix with examples.

5.
 - a. Use properties of determinant to show that :

$$\begin{vmatrix} a & 1 & b+c \\ b & 1 & c+a \\ c & 1 & a+b \end{vmatrix} = 0$$
 - b. Find the limit if it exists $\lim_{x \rightarrow 2} \frac{x-2}{|x-2|}$
 - c. Find the inverse of: $\begin{bmatrix} 3 & 2 \\ -1 & 6 \end{bmatrix}$

Group B [5×2×4=40]

6.
 - a. Solve: $4 + 3x - x^2 > 0$
 - b. Evaluate: $\lim_{x \rightarrow a} (a-x) \tan \frac{\pi x}{2a}$

7.
 - a. A function is defined as follows:

$$f(x) = \begin{cases} \frac{2x^2 - 18}{x-3} & \text{for } x \neq 3 \\ k & \text{for } x = 3 \end{cases}$$
 Find the value of k so that $f(x)$ is continuous at $x = 3$.
 - b. If $\sin y = x \sin(a+y)$ then prove that $\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$

8.
 - a. Show that the rectangle of largest possible area for a given perimeter is square.
 - b. Evaluate: $\int \sec^6 x \, dx$

9. a. Find the square roots of $-7 + 24i$.
 b. Form the equation whose roots are reciprocals of the roots of $ax^2 + bx + c = 0$.

10. a. Show that:
$$\begin{vmatrix} 1+x & 1 & 1 \\ 1 & 1+y & 1 \\ 1 & 1 & 1+z \end{vmatrix} = xyz \left(\frac{1}{x} + \frac{1}{y} + \frac{1}{z} + 1 \right).$$

- b. Applying Cramer's rule. Solve the following system of linear equation.
 $x + y + z = 1$ $x + 2y + 3z = 4$ $x + 3y + 7z = 13$

Group C

[5×6=30]

11. What condition make a function to have it's inverse? Prove that a function $f : R \rightarrow R$ defined by $f(x) = 5x + 2$ is one to one and onto also find $f^{-1}(x)$.
12. Find from first principal the derivative of $\tan(3x - 4)$ also find the derivative of $e^{\sin x^2}$.
13. Water flows into an inverted conical vessel at the rate of $24m^3 / \text{min}$. When the depth of water is $4m$, how fast is the level of water rising, assuming that the height of the vessel is $8m$ and the radius at the top is $2m$.
14. State De-Moivre's theorem hence solve $z^6 = 1$.
15. Solve the system:

$$2x - 3y + 4z = 4$$

$$3x + 4y - z = 5$$

$$4x + y + z = 6$$
 By using row-equivalent matrix method.



Second Term Exam-2071

Grade: XI
Time: 3:00 hrs.

Subject: Basic Mathematics

F.M.:100
P.M.:40

Set B

Group A

[5×3×2=30]

1.
 - a. If p and q are two simple statements then prove that $(p \wedge q) \wedge \sim (p \vee q)$ is a contradiction.
 - b. Prove that a function $f : N \rightarrow N$ defined by $f(x) = 2x$ is not onto function.
 - c. Prove that : $A - B = \overline{B - A}$

2.
 - a. Evaluate: $\lim_{x \rightarrow a} \frac{\sqrt{x} - \sqrt{a}}{\tan(x - a)}$.
 - b. A function $f(x)$ is defined as follows

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 3 \\ 2kx & \text{for } x \geq 3 \end{cases}$$
 Find the value k . So that $f(x)$ is continuous at $x = 3$.
 - c. Find the derivative of $y = \cos[\sin(\log x)]$.

3.
 - a. Find the interval in which the function $f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing or decreasing.
 - b. Evaluate: $\int \tan^3 x \, dx$
 - c. Evaluate: $\int \frac{1}{1 + e^{-x}} \, dx$

4.
 - a. If the equation $(1 + m^2)x^2 + 2mcx + c^2 - a^2 = 0$ has equal roots. Show that $c^2 = a^2(1 + m^2)$.
 - b. Express the complex number $i - \sqrt{3}$ in the polar form
 - c. Define a triangular matrix with example. What is difference between upper and lower triangular metrics?

5.
 - a. Use properties of determinant to show that:

$$\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix} = 0$$
 where w is an imaginary cube root of unity.
 - b. Find the limit if it exist $\lim_{x \rightarrow 2} \frac{|x - 2|}{x - 2}$.
 - c. Solve by inversion method: $x + y = 2$ $x - y = 0$

Group B

[5×2×4=40]

6.
 - a. Define the absolute value of a real number. If a is any positive real number and $x \in R$, Prove that $|x| < a \Leftrightarrow -a < x < a$.
 - b. Evaluate : $\lim_{x \rightarrow 0} \frac{(a + x)\sec(a + x) - a \sec a}{x}$

7.
 - a. If $f(x) = \begin{cases} 3ax + b & \text{for } x > 1 \\ 1 & \text{for } x = 1 \\ 5ax - 4 & \text{for } x < 1 \end{cases}$ is continuous at $x = 1$, find the Value of a and b .
 - b. If $x = a(t - \sin t)$ and $y = a(1 + \cos t)$, then prove that $\frac{d^2 y}{dx^2} = \frac{1}{4a} \operatorname{cosec}^4\left(\frac{t}{2}\right)$.

8. a. A man who has 144 meters of fencing material wishes to enclose a rectangular garden. Find the maximum area he can enclose.
- b. Evaluate: $\int e^{x+1/x} \left(1 - \frac{1}{x^2}\right) dx$.
9. a. Applying Cramer's rule. Solve the following system of linear equation.
 $x + y + z = 6$
 $x - y + z = 2$
 $x + y - z = 0$
- b. Prove that: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$
10. a. Find the square roots of $-8 - 6i$
- b. Find the equation whose roots are reciprocal to the roots $x^2 - x + 1 = 0$.

Group C

[5×6=30]

11. What is function? Find the domain and Range of the $f(x) = \sqrt{6 - x - x^2}$.
12. Find the from first principle of $\cot(2x - 3)$. Also find the derivative of $e^{\sin(\log x)}$.
13. Water flows into an inverted conical tank at the rate of $27 \text{ ft}^3 / \text{min}$. When the depth of water is 2 ft , how fast the level rising? Assume that the height of the tank is 4 ft and the radius of the top is 1 ft .
14. Express a Complex number $x + iy$ in a polar form. State De-Moivre's theorem. Use De- Moiver's theorem to find the cube roots of unity.

15. Solve the system
 $3x_1 + 4x_2 - x_3 = 5$
 $4x_1 + x_2 + x_3 = 6$
 $2x_1 - 3x_2 + 3x_3 = 4$
 By using row-equivalent matrix method.