GLOBAL COLLEGE OF MANAGEMENT

Pre-Board Exam – 2071

Grade: XI Subject: Basic Mathematics F.M.: 100

Time: 3 hrs. P.M.: 40

Set A

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group-A **[10×2×3 = 60]**

1. (a) Define disjunction of two statements. Prepare the truth table for the compound statement $\sim[p\wedge(\sim q)]$.
   (b) Let $A=\{1, 2, 3, 4\}$. Find the relation on A satisfying the condition $x+y\leq 4$. Is this relation a function? Give reason.
   (c) Test the periodicity and the symmetricity of the function $y=\cos\pi x$

2. (a) Solve the equation: $3\tan^2 x-1=0$.
   (b) Using the mathematical induction prove that $2+4+6.......+2n=n\,(n+1)$
   (c) If $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}$. Show that $A^T.A=I$

3. (a) Solve by inverse matrix method
   $3x-3y=11$
   $9x-2y=5$
   (b) Represent the complex number $-4+i4\sqrt{3}$ in polar form.
   (c) For what value of p will the equation $5x^2-px+45=0$ have equal roots?

4. (a) Determine whether the points $(2,1)$ *and* $(-2, 3)$ are on the same side of the line $3x+7y-5=0$ or not.
   (b) Find the condition for the line $lx+my=n$ to be tanget to the circle $x^2+y^2=a^2$.
   (c) Evaluate: $\lim_{x\to\infty}\left(\sqrt{2+x}-\sqrt{2x}\right)$

5. (a) Find the derivative of : $\tan[\sin(ax-b)]$
   (b) Show that a function $f(x)=5x-\frac{2}{x}$ is increasing for all $x\in R$ except $x=0$.
   (c) Evaluate: $\int\frac{\cos\sqrt{x}}{\sqrt{x}}dx$

Group-B **[8×5 = 40]**

6. (a) If A, B and C are the subsets of universal set $\cup$. Prove that $A-(B\cup C)=(A-B)\cap(A-C)$.

   OR

   Define a absolute value of a real number. If a is any positive real number and $x\in R$. Prove that $|x|<a\Leftrightarrow -a<x<a$.
   (b) Sketch the graph of $y=x^2-6x+9$ indicating it's different characterstics.

7. (a) If $\cos^{-1}x+\cos^{-1}y+\cos^{-1}z=\pi$. Prove that $x^2+y^2+z^2+2xyz=1$.

   OR

   State and prove cosine law in any triangle ABC.
   (b) Prove that $\begin{vmatrix}a-b-c & 2a & 2a\\ 2b & b-c-a & 2b\\ 2c & 2c & c-a-b\end{vmatrix}=(a+b+c)^3$

8. (a) Applying Cramer's rule or row equivalent method solve the system of linear equations:

$$2x - y + z = -1$$
$$x - 2y + 3z = 4$$
$$4x + y + 2z = 4$$

(b) If the roots of the equations $lx^2 + nx + n = 0$ be in the ratio $p:q$. Prove that $\sqrt{\frac{p}{q}} + \sqrt{\frac{q}{p}} + \sqrt{\frac{n}{l}} = 0$

9. (a) If tangents are drawn from the points $(4, -2)$ to the circle $x^2 + y^2 = 10$ find their equations. Also show that they are at a right angle.

(b) Evaulate: $\lim_{x \to \theta} \frac{x \sin\theta - \theta \sin x}{x - \theta}$

OR

A function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} 2x+3 & \text{for} \quad x < 1 \\ 4 & \text{for} \quad x = 1 \\ 6x-1 & \text{for} \quad x > 1 \end{cases}$$

Is the function continuous at $x = 1$. If not, how can make it continuous?

10. (a) Find the derivates of $\cos 4x$ from the first principle.

(b) Find the area of the curves $\frac{x^2}{4} + \frac{y^2}{9} = 1$ be integration method.

Group-C **[8×5 = 40]**

11. Define domain and range of a function. Find the domain and the range of the function $f(x) = \sqrt{6 - x - x^2}$

12. Prove that $A.M$ $G.M$ and $H.M$ between the unequal positive quantities satisfy, G.M.>H.M. Also if G_1 and G_2 are two geometric means between b and c, and a is their arithmetic mean, then show that $G_1^3 + G_2^3 = 2abc$.

13. Derive the formula for the length of the perpendicular from a point (x_1, y_1) to the line $x\cos\propto + y\sin\propto = p$ also, find the distance between the parallel lines $3x + 5y = 11$ and $3x + 5y = -23$

OR

Find the condition that the general equation of second degree may represent a line pair. If the equation $ax^2 + 2hxy + by^2 + 2gx + 2ty + c = 0$ represents a pair of parallel lines, prove that $\frac{a}{h} = \frac{h}{b} = \frac{g}{f}$.

14. Define conjugate of complex number. Solve $z^4 = 1$.By using De-Moiver's theorem.

15. Two concentric circles are expanding in a such a way that the radius of the inner circle is increasing at the rate of *10 cm/sec.* and that of the outer circle is at the rate of *7 cm/sec.* At a certain time the radius of the inner and the outer circles and respectively *24 cm*. and *30 cm*. At that time, is the area between the circle increasing and decreasing? How fast.

OR

Find the interval where the given function $f(x) = x^4 - 8x^3 + 18x^2 - 24$ is concave upward, also find the maximum area of rectangular plot of land which can br enclosed by a rope of length 60 meters.

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Set B

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group-A **[10×2×3 = 60]**

1. (a) Define tautology. Construct a truth table for the compound statement $\sim [p \vee (\sim q)]$

(b) If $A = \{1, 2, 3\}$ find the relation on A satisfying the condition $x + y < 4$. Is the relation a function? Give reason.

(c) Test the periodicity and the symmetricity of the function $y = \sin x$

2. (a) Solve the equation: $\sin x + \sqrt{3}\cos x = \sqrt{2}$.

(b) Using the mathematical induction prove that $1+3+5+\text{..........}.+(2n+1) = n^2$.

(c) Solve for x: $\begin{vmatrix} x & 3 & 3 \\ 3 & 3 & x \\ 2 & 3 & 3 \end{vmatrix} = 0$

3. (a) Solve by inverse matrix method: $3x - 2y = 8 \quad 5x + 3y = 7$.

(b) Represent the complex number $\left(-1 + i\sqrt{3}\right)$ in the polar form.

(c) If $x^2 - 2mx + 18m - 15 = 0$ has equal roots. Find the value of m.

4. (a) Show that the points $(1, 2)$ and $(2, -3)$ lie on the opposite side of the line $5x - 2y - 3 = 0$.

(b) Find the condition for the line $y = mx + c$ to be tanget to the circle $x^2 + y^2 = a^2$

(c) Evaluate: $\lim_{x\to\infty} \sqrt{x}\left(\sqrt{x} - \sqrt{x-a}\right)$

5. (a) Find the derivative of $\sqrt{\cos\sqrt{x}}$

(b) Show that the function $f(x) = 4x - \frac{9}{x} + 6$ is increasing for all $x \in R$ expect at $x = 0$.

(c) Evaluate: $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

Group-B **[8×5 = 40]**

6. (a) State and prove De-morgan's law in set theory.

OR

Define absolute value of real number, also for any $x, y \in R$. Prove that

i. $|x + y| \le |x| + |y|$

ii. $|x - y| \ge |x| - |y|$

(b) Sketch the graph of the function $y = (x-1)(x-2)(x-3)$ indicating it's different characteristics.

7. (a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$. Prove that $x + y + z = xyz$.

OR

State and prove sine law in any triangle ABC.

(b) Prove that $\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$

8. (a) Applying Cramer's rule or row equivalent method. Solve the system of linear equations.

$$4x-3y+z=1$$
$$x+4y-2z=10$$
$$2x-2y+3z=4$$

(b) If the equation $x^2+px+q=0$ and $x^2+qx+p=0$ have a common root. Prove that either $p=q$ or $p+q+1=0$.

9. (a) Prove that two circles $x^2+y^2+2ax+c^2=0$ and $x^2+y^2+2by+c^2=0$ touch if $\frac{1}{a^2}+\frac{1}{b^2}=\frac{1}{c^2}$

(b) Evaluate: $\lim_{x\to\theta}\frac{x\cot\theta-\theta\cot x}{x-\theta}$.

OR

A function $f(x)$ is defined as follows:

$$f(x)=\begin{cases}2x-3 & \text{for} \quad x<2\\ 2 & \text{for} \quad x=2\\ 3x-5 & \text{for} \quad x>2\end{cases}$$

Is $f(x)$ continuous at $x=2$. If not, how can $f(x)$ be made continuous at $x=2$.

10. (a) Find the derivative of $\sin 4x$ from the first principles.

(b) Find the area between the curves $y^2=4ax$ and $x^2=4ay$.

Group-C **[5×6 = 30]**

11. Define Domain and Range of the function. Find the Domain and Range of the function $f(x)=\sqrt{21-4x-x^2}$.

12. If G_1 and G_2 are two geometric means between b and c and a is their arithimic mean, then show that $G_1^3+G_2^3=2abc$. Also prove that A.M. and G.M. and H.M. between unequal positive quantities satisfy A.M.>G.M.

13. The origin is a corner of the square and two of it's sides are $y+2x=0$ and $y+2x=3$. Find the equations of other two sides. Also prove that the distance between two parallel lines $y=mx+c_1$ and $y=mx+c_2$ is $\frac{|c_2-c_1|}{\sqrt{1+m^2}}$.

OR

Prove that the product of the perpendicular from (α,β) to the lines given by $ax^2+2hxy+by^2=0$ is $\frac{a\alpha^2+2h\alpha\beta+b\beta^2}{\sqrt{(a-b)^2+4h^2}}$. Also find the acute angle of bisector between the lines $4x+3y-7=0$ and $24x+7y-31=0$

14. Define absolute value of complex number and solve $Z^6=1$.By using De-Moiver's theorem.

15. Find the interval where given function $f(x)=x^4-8x^3+18x-24$ is concave downward. Also, show that the rectangle of largest area for a given perimeter is square.

OR

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate *8cm/sec.* and that of the outer circle at the rate of *5cm/sec*. At a certain instant the radii of the inner and outer circles are respectively *24cm* and *30cm*. At what rate does the area between the two circles changes.