



Second Term Exam – 2070

Grade: XI
Time: 3 hrs.

Subject: Mathematics

F.M.:100
P.M.: 40

Set A

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group ‘A’ [5×3×2 = 30]

1. (a) Define the disjunction of two statements. Construct the truth table for the statement $\sim p \wedge q$.
(b) Let $a, b, c \in \mathbb{R}$, if $a < b$, using the properties of real number prove that $a - c < b - c$.
(c) Prove that: $\tan^{-1} \left(\frac{1 + \cos x}{\sin x} \right) = \frac{\pi}{2} - \frac{x}{2}$
2. (a) In any ΔABC , prove that: $\frac{\cos B - \cos C}{\cos A + 1} = \frac{c - b}{a}$
(b) Show that the matrices $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}$ and $\begin{pmatrix} 2 & -1 \\ 5 & -3 \end{pmatrix}$ are inverse of each other.
(c) Solve by Cramer's rule: $3x + 2y = 8, 4x + y = 9$
3. (a) Find the square root of $2 + 2\sqrt{3}i$
(b) Prove that the roots of the equation $(x - a)(x - b) = k^2$ are real for all values of $k \in \mathbb{R}$.
(c) Find the value of k so that the line whose equation is $x + y = k$ will form a triangle with the coordinate axes whose area is 32 sq. units.
4. (a) Show that the function $f(x) = \frac{1}{1-x}$ is continuous at $x \neq 1$?
(b) Evaluate: $\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$

- (c) Find $\frac{dy}{dx}$ when $y = \frac{1}{\sec x - \tan x}$
5. (a) Show that the rectangle of largest possible area, for a given perimeter, is a square.
(b) Integrate: $\int \frac{e^{2x}}{1 + e^x} dx$
(c) Integrate: $\int \frac{x+2}{\sqrt{x+1}} dx$

Group ‘B’ [5×2×4 = 40]

6. (a) Define absolute value of a real number. For any two real numbers x and y , prove that: $|x + y| \leq |x| + |y|$.
(b) Solve: $\tan \theta + \sec \theta = \sqrt{3}$
7. (a) Two sides of a triangle are $\sqrt{3} + 1$ and $\sqrt{3} - 1$ and the included angle is 60° , solve the triangle.
(b) Find the value of: $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$
8. (a) Solve by row - equivalent method:
 $9y - 5x = 3, x + z = 1$ and $z + 2y = 2$.
(b) Obtain the equations of straight lines through the point $(\frac{1}{\sqrt{3}}, 1)$ whose perpendicular distance from origin unity.
9. (a) Evaluate: $\lim_{x \rightarrow \theta} \frac{x \cot \theta - \theta \cot x}{x - \theta}$
(b) A function $f(x)$ is defined as follows:
$$f(x) = \begin{cases} 2x+1 & \text{for } x < 1 \\ 2 & \text{for } x = 1 \\ 3x & \text{for } x > 1 \end{cases}$$

Is the function continuous at $x = 1$? If not, can it be made continuous at $x = 1$?
10. (a) Find, from definition, the derivative of $\frac{1}{\sqrt{x+2}}$
(b) Integrate: $\int (7x + 9)\sqrt{4x + 1} dx$

Group 'C' [5×6 = 30]

11. Define domain and range of a function. Find domain and range of the function $y = \sqrt{x^2 - 2x - 8}$
12. Define De Moivre's theorem. Use its application to find the value of $z^6 = 1$.
13. If the quadratic equations $ax^2 + bx + c = 0$ and $bx^2 + cx + a = 0$ have a common root, then either $a + b + c = 0$ or $a = b = c$.
14. Find the local maximum and minimum values of the function $f(x) = 12x^5 - 45x^4 + 40x^3 + 6$. Also find the points of inflection (if they exist).
15. Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate of 8cm/sec and that of the outer circle at the rate of 5cm/sec. At a certain instant the radii of the inner and outer circle are respectively 24 cm and 30 cm. At what rate does the area between the two circles?



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Group ‘A’ [5×3×2 = 30]

1. (a) Define conjunction of two statements. Construct the truth table for the statement $\sim q \vee p$.
(b) Let $a, b, c \in \mathbb{R}$, If $a < b$ and $b < c$, using the properties of real number prove that $a < c$.
(c) Find the value of $\text{Arc sin } t - \text{Arc cos } (-t)$
2. (a) In any triangle ABC, if $\cos B = \frac{\sin A}{2\sin C}$, show that the triangle is isosceles.
(b) Given a matrix $\begin{pmatrix} 3 & -1 \\ 5 & -2 \end{pmatrix}$, find a matrix $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$ such that they are inverse of each other.
(c) Solve by row-equivalent method: $3x - 2y = 8; 5x + 3y = 7$
3. (a) Find the multiplicative inverse of $(2 + i)^2$
(b) For what value of a will the equation $x^2 - (3a - 1)x + 2(a^2 - 1) = 0$ have equal roots?
(c) Show that two of the three points $(0, 0)$, $(2, 3)$ and $(3, 4)$ lie on one side and the remaining on the other side of the line $x - 3y + 3 = 0$.
4. (a) Why the function $f(x) = \text{Sin } \frac{1}{x}$ is not continuous at $x = 0$?
(b) Evaluate: $\lim_{x \rightarrow 0} \frac{\tan 2x - x}{3x - \sin x}$

(c) Find $\frac{dy}{dx}$ Where $y = \tan^{-1} \left(\frac{2x}{1-x^2} \right)$

5. (a) Divide 60 into two parts such that the sum of the squares of the two parts is maximum.
(b) Integrate: $\int \frac{1+e^x}{1+e^x} dx$.
(c) Integrate $\int \text{Sec } x \, dx$.

Group ‘B’ [5×2×4 = 40]

6. (a) If $x \in \mathbb{R}$ and a is any positive real number, Prove that:
 $|x| < a \Leftrightarrow -a < x < a$.
(b) Solve: $\sqrt{2} \sec \theta + \tan \theta = 1$.
7. (a) In a triangle ABC, If $a = 2b, A = 3B$, find the angles of the triangle.
(b) Show that: $\begin{vmatrix} x^2 + 1 & xy & xz \\ xy & y^2 + 1 & yz \\ xz & yz & z^2 + 1 \end{vmatrix} = 1 + x^2 + y^2 + z^2$
8. (a) Solve by Cramer's rule:
 $x + 2y + 3z = 6, 2x + 4y + z = 7$ and $3x + 2y + 9z = 14$.
(b) P and Q are two points on the line $x - y + 1 = 0$ and are at distance 5 units from the origin. Find the area of the triangle OPQ.
9. (a) Evaluate: $\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$
(b) A function $f(x)$ is defined as follows:
$$f(x) \begin{cases} 3 + 2x & \text{for } -\frac{3}{2} \leq x < 0 \\ 3 - 2x & \text{for } 0 \leq x < \frac{3}{2} \\ -3 - 2x & \text{for } x \geq \frac{3}{2} \end{cases}$$

Show that $f(x)$ is continuous at $x = 0$ & discontinuous at $x = \frac{3}{2}$.

10. (a) Find, from definition, the derivative of $\frac{1}{\sqrt{3x-2}}$
(b) Integrate: $\int \tan^3 x \sec^4 x \, dx$

Group 'C' [5×6 = 30]

11. Define domain and range of a function. Find domain and range of the function $y = \sqrt{21 - 4x - x^2}$.
12. Define complex number, Express a complex number into polar form. Using De'Moivre's theorem, find the cube roots of unity.
13. Find the local maxima and minima and also the point of inflection (if exists) of the function $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval $(-1, 2)$. Also examine whether the function is increasing or decreasing at $x = 0$.
14. A point is moving along the curve $y = 2x^3 - 3x^2$ in such a way that its x-coordinate is increasing at the rate of 2 cm/sec, Find the rate at which the distance of the point from the origin is increasing when the point is at $(2, 4)$.
15. Prove that if the equations $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root, their other roots will satisfy $x^2 + ax + bc = 0$.