GLOBAL COLLEGE OF MANAGEMENT

Pre Board Exam – 2070

Grade: XI Subject: Mathematics F.M.: 100

Time: 3 hrs. P.M.: 35

Set - A

Students are required to give their answers in their own words as far as practicable. The figures in the margin indicate full marks. Omissions in essential parts will loss in marks.

Group 'A'

Attempt **all** the questions. [5×3×2= 30]

1. a) Define inverse of a compound statement. If p and q are any two statements, form a truth value table of $p \wedge (\sim q)$.
   b) Let $f : R \to R$ and $g : R \to R$ be defined by $f(x) = 3x - 1$ and $g(x) = x^2$, find i) $(3f - 2g)(x)$ ii) $(fg)(x)$
   c) Test the periodicity of the function f(x) = cos 2x and find its period.

2. a) Solve: sec x.tan x = $\sqrt{2}$
   b) If α and β are the complex cube roots of unity prove that:
   $\alpha^4 + \beta^4 + \alpha^{-1}.\beta^{-1} = 0$.
   c) Find the value of $9^{\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+.....to\ \infty}$

3. a) Define singular and nonsingular matrix with one example of each .
   b) If the roots of the equation $x^2 - 8x + a^2 - 6a = 0$ are real and distinct, then find all possible value of a.
   c) Apply Cramer's rule to solve the following:
   $3x + \frac{4}{y} = 10; -2x + \frac{3}{y} = -1$

4. a) Does the equation $2x^2 + 7xy + 3y^2 - 4x - 7y + 2 = 0$ represents a pair of straight lines?
   b) Find the equation for the circle which passes through two points on the x-axis which are at distance 4 from the origin and where radius is 5
   c) Find the value of: $\lim_{x \to 64} \frac{\sqrt[6]{x} - 2}{\sqrt[3]{x} - 4}$

5. a) If $f(x) = mx + c$ and $f(0) = f'(0) = 1$, What is $f(2)$?
   b) A 2.5 m. ladder leans against a vertical wall. If the top slides downward at the rate of 12 cm/sec, find the speed of the lower end when it is 2m. from the wall.
   c) Evaluate: $\int \sin^2 2x \, dx$.

Group 'B'

Attempt **all** the questions. [5×2×4 = 40]

6. a) Prove that a function f : R → R defined by f (x) = 3x – 1 is bijective.

   OR

   Using properties of real number, prove that
   (i) $a > b$ and $c < 0 \Rightarrow ac < bc$
   (ii) $a > b$ and $c > 0 \Rightarrow \frac{a}{c} > \frac{b}{c}$
   b) Indicating all characteristics draw the graph of $y = f(x) = \sin x$.

7. a) In any triangle ABC, prove that:
   $(a - b)^2 \cos^2 \frac{C}{2} + (a + b)^2 \sin^2 \frac{C}{2} = c^2$

   OR

   If a = 2b , A = 3B , find the angles of the triangle.

   b) Show that $\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (c+a)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix} = 2abc(a+b+c)^3$

8. a) Apply row equivalent method to solve:
3x + 5y = 2; 2x – 3z = -7; 4y + 2z = 2

b) Prove that if the equations $x^2 + bx + ca = 0$ and $x^2 + cx + ab = 0$ have a common root, their other roots will satisfy $x^2 + ax + bc = 0$.

9. a) Find the conditions for the two circles $x^2 + y^2 = a^2$ and $(x - c)^2 + y^2 = b^2$ to touch i) externally ii) internally.

b) Prove that for any positive and negative integer n,

$$\lim_{x \to a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

OR

Find the point of discontinuity of the following function

$f(x) = \frac{3x-1}{x^3 - 5x^2 + 6x}$. Also verify the discontinuity.

10. a) Find the derivative by the definition: $\frac{ax+b}{\sqrt{x}}$

b) Integrate: $\int \text{cosec}^3 x \, dx$

OR

Find the area enclosed by the X axis and the curve
y = (x – 1)(x – 2)(x – 3).

Group 'C'

Attempt **all** the questions. [5×6 = 30]

11. Define the domain and range of a function. Find the domain and range of the function $f(x) = \sqrt{24 - 2x - x^2}$.

12. Prove by the method of induction that

$$1 \cdot 3 + 2 \cdot 4 + 3 \cdot 5 + \ldots\ldots\ldots + n \cdot (n+2) = \frac{n(n+1)(2n+7)}{6}$$

OR

Sum to n terms of the series $1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots\ldots\ldots$

13. Show that the two straight lines $x^2(\tan^2\theta + \cos^2\theta) - 2xy \tan\theta + y^2 \sin^2\theta = 0$ makes with x-axis angles such that the difference of their tangent is 2.
Prove that the equation of its angle bisector is
$(x^2 - y^2)\sin 2\theta + xy(2 - \sin^2 2\theta) = 0$

OR

The origin is a corner of a square and two of its sides are given by $2x + y = 0$ and $2x + y = 3$. Find the equation of other two sides. Also find the locus of the point which moves so that it is equidistant from the two lines $3x + 4y = 11$ and $12x - 5y = 2$.

14. Define complex number and its conjugate. State De'Moivres theorem. If z_1 and z_2 are two complex numbers, then show that $|z_1 + z_2| \le |z_1| + |z_2|$.

OR

Define the k^{th} root of a complex number. Prove that if $z = r(\cos\theta + i\sin\theta)$ then the distinct k^{th} roots of Z given by

$$Z_n = \sqrt[k]{r}\left(\text{Cos}\frac{2n\pi + \theta}{k} + i\sin\frac{2n\pi + \theta}{k}\right) \text{ for n= 0, 1, 3… k–1.}$$

15. A point is moving along the curve $y = 2x^3 - 3x^2$ in such a way that its x- coordinate is increasing at the rate of 4ft/sec. Find the rate of which the distance of the point from the origin is increasing when the point is at (2,4) .

OR

A window is in the form of a rectangle surmounted by a semi-circle. If the total perimeter is 9 meters, find the radius of the semi-circle for the greatest window area.

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Set - B

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Group 'A'

Attempt **all** the questions. [5×3×2 = 30]

1. a) Define contrapositive of a compound statement. If p and q are any two statements, form a truth value table of $(\sim p) \vee q$.
   b) Check whether the function f : [-2,2] → R given by f (x) = x^2 is one to one and find the range.
   c) Test the periodicity of the function f(x) = sin (2x + 3) and find its period.

2. a) Solve: $\cot x + \tan x = 2$.
   b) Find the square root of $-i + \sqrt{3}$ i .
   c) If H be the H.M. between a and b, prove that $(H - 2a)(H - 2b) = H^2$.

3. a) Apply row equivalent method to solve: $5x - 4y = -3$ and $7x + 2y = 49$.
   b) If the difference between the roots of $x^2 - px + q = 0$ be 2, find the relation between p and q.
   c) If $A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}$, find x and y such that xA + yI = 0, where I and O are 2×2 identity and null matrix.

4. a) Find the separate equations of the lines represented by $x^2 + 2xy \sec\theta + y^2 = 0$.
   b) Prove analytically that the angle in a semi-circle is a right angle.
   c) Evaluate: $\lim_{\theta \to \frac{\pi}{4}} \frac{\cos\theta - \sin\theta}{\theta - \frac{\pi}{4}}$

5. a) Find $\frac{dy}{dx}$ of $y = \tan^{-1} \sqrt{\frac{1 - \sin x}{1 + \sin x}}$
   b) Find indefinite integrals: $\int \frac{1}{1 + e^x}\, dx$
   c) A ball thrown vertically up reaches a height of $2 + 30t - 8t^2$ in t sec. Find maximum height will it go?

Group 'B'

Attempt **all** the questions. [5×2×4 = 40]

6. a) If $x \in \Re$ and a be any positive real number then $|x| < a$ iff $-a < x < a$.

   OR

   Solve the inequality $\frac{x+2}{2-3x} \geq 0$ and draw the graph.
   b) Sketch the graph of the function f(x) = (x-1) (x-2) (x-3) indicating its specific characteristics.

7. a) Solve: $3\tan^{-1}\frac{1}{2+\sqrt{3}} - \tan^{-1}\frac{1}{x} = \tan^{-1}\frac{1}{3}$

   OR

   In any triangle ABC, prove that
   $\frac{a^2 \sin(B-C)}{\sin B + \sin C} + \frac{b^2 \sin(C-A)}{\sin C + \sin A} + \frac{c^2 \sin(A-B)}{\sin A + \sin B} = 0$
   b) Prove that:
   $$\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 2bxy + cy^2)$$

8. a) Apply matrix method to solve: x + y – z = 1; y + z = 2; x – y = 0.

b) What are the relations between the roots and the coefficients of the quadratic equation $ax^2 + bx + c = 0$? Obtain the condition so that the equations $a_1x^2 + b_1x + c_1 = 0$ and $a_2x^2 + b_2x + c_2 = 0$ has one root common.

9. a) Find the equation of the circle whose centre is at the point (h, k) and which passes through the origin and prove that the equation of the tangent at the origin is $hx + ky = 0$.

b) Find the limiting values of : $\lim_{y\to 0} \frac{(x+y)\sec(x+y) - x\sec x}{y}$

OR

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 3+2x & for\ \frac{-3}{2} \le x < 0 \\ 3-2x & for\ 0 \le x < \frac{3}{2} \\ -3-2x & for\ x \ge \frac{3}{2} \end{cases}$$

Show that the function is continuous at x = 0 and discontinuous at $x = \frac{3}{2}$.

10. a) Find the derivative of $x + \sqrt{x}$ from first principles.

b) $\int \frac{\sin x \cos x dx}{a\cos^2 x + b\sin^2 x} dx$

OR

Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Group 'C'

Attempt **all** the questions. [5×6 = 30]

11. Define even and odd function, Find the domain and range of the function defined in the set of the real numbers; $y = \sqrt{21 - 4x - x^2}$

12. Find the sum of n terms to the series $3.1^2 + 4.2^2 + 5.3^2 + \ldots\ldots\ldots$

OR

Using the principle of mathematical induction, prove that:

$1^2 + 3^2 + 5^2 + \ldots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$

13. Derive the formula for the length of the perpendicular from a point (x_1, y_1) to a line $x\cos\alpha + y\sin\alpha = p$. Also, find the distance between the parallel lines 5x – 12y + 8 = 0 and 10x + 24y -3 = 0.

OR

Show that the equation to the pair of lies through the origin and perpendicular to $ax^2 + 2hxy + by^2 = 0$ is $bx^2 - 2hxy + ay^2 = 0$.

Also prove that $6x^2 - 5xy - 6y^2 + 14x + 5y + 4 = 0$ represent a pair of perpendicular lines.

14. Express a complex number in polar form. State De-Moivre's theorem. Using De-Moivre's theorem find the cube roots of unity.

15. Show that the height of a closed cylinder of given surface and maximum volume is equal to diameter of its base.

OR

A ship leaves a port at noon and travels east at the rate of 9 km/hr. Another ship leaves the same port after one hour and sails south at the rate of 12 km/hr. How fast are they separating at 2 PM?

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